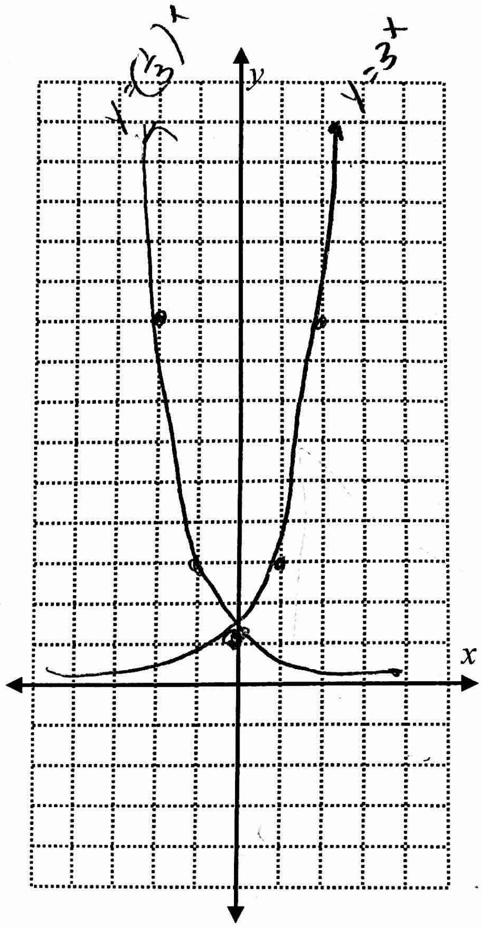
Name: ______________________________ Date: ____________________

REVIEW OF UNIT #4
COMMON CORE ALGEBRA II



1. a. On the accompanying grid, sketch the graphs of $y=3^x$ and $y=\left(\frac{1}{3}\right)^x$.

   b. Give the domain and range for each function.

x	y
0	1
1	3
2	9

x	y
0	1
1	.3
2	.2

2. Which of the following is *not* in the domain of $y=3^x$?

   (1) -1 (2) 0 (3) 1 (4) they are all in the domain

3. Which of the following is *not* in the range of $y=4^x$?

   (1) 0 (2) 5 (3) 1 (4) they are all in the range

4. Express each of the following as a fraction in simplest form:

   a. If $f(x)=2^x$, find $f(-4)$.

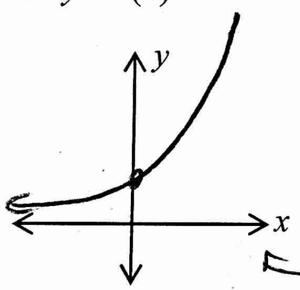
   $2^{-4} = \frac{1}{16}$

   b. If $f(x)=8^x$, find $f\left(-\frac{1}{3}\right)$.

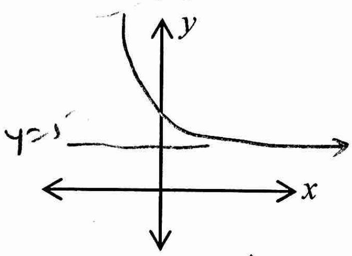
   $8^{-\frac{1}{3}} = \frac{1}{2}$

5. Sketch the graphs of the following on the accompanying axis: (label y-int)

   a. $y=2(4)^x$



   b. $y=4\left(\frac{1}{2}\right)^x+5$



6. Give the equations of the asymptotes of the equations from #5.

   $y=0$ $y=5$

7. Evaluate each of the following:

   a. $64^{\frac{2}{3}}$ b. $81^{\frac{1}{4}}$ c. $36^{\frac{3}{2}}$ d. $243^{-\frac{3}{5}}$ e. $81^{-\frac{3}{4}}$

   16 3 216 $\frac{1}{27}$ $\frac{1}{27}$

8. Solve for x: $x^{\frac{2}{3} \cdot 3/2} = 36^{3/2}$

   216

$\frac{1}{\sqrt{x^3}}$

9. Written without rational or negative exponents, $x^{-\frac{3}{2}}$ is equal to

(1) $-\frac{3x}{2}$ (2) $\frac{1}{\sqrt{x^3}}$ (3) $\frac{1}{\sqrt[3]{x^2}}$ (4) $-\frac{1}{\sqrt{x}}$

10. Which of the following is equivalent to $y^{\frac{5}{2}}$?

$\sqrt{y^5}$

(1) $\left(\sqrt{y}\right)^5$ (2) $\sqrt[5]{y}$ (3) $\left(\sqrt[5]{y}\right)^2$ (4) $5\sqrt{y}$

11. The monomial $9x^{\frac{5}{2}}$ can be rewritten equivalently as

$9\sqrt{x^5}$

(1) $9\sqrt{x^5}$ (2) $\sqrt[5]{9x^2}$ (3) $3\sqrt{x^5}$ (4) $243\sqrt{x^5}$

$\frac{1}{2\sqrt[3]{x}}$

12. Expressed in an equivalent manner, $(8x)^{-\frac{1}{3}}$ is

(1) $\frac{1}{2\sqrt[3]{x}}$ (2) $\frac{1}{8\sqrt{x}}$ (3) $\frac{\sqrt{x}}{2}$ (4) $\frac{1}{\sqrt[3]{2x}}$

Lesson 2 - writing exp equat.

13. Emily purchased a new car for \$30,000, but when she went to sell it in 5 years it was only worth \$10,500.

Find an exponential function in the form $V(t) = a(b)^t$ to model the value, V, of the car at any time t.

$a = 30{,}000$

$(0, 30{,}000)$ $(5, 10{,}500)$

$V(t) = 30000(.81)^t$

$\frac{10{,}500}{30000} = \frac{30000\, b^5}{30000}$

$.35^{1/5} = b^{5 \cdot 1/5}$ $= .8106$

14. Find the equation of the exponential of the form $y = a(b)^x$ that passes through $(0, 150)$ and $(2, 60)$.

$y = ab^x$ $y = ab^x$ $a = 150$

$\frac{60}{150} = \frac{150\, b^2}{150}$

$.4 = b^2$

$.632455532 = b$

$y = 150(.63)^x$

15. Find the equation of the exponential of the form $y = a(b)^x$ that passes through (3, 18) and (7, 389). Round a to the *nearest integer*, and b to the *nearest hundredth*.

$y = ab^x$
$18 = ab^3$

$y = ab^x$
$389 = ab^7$

$\frac{389 = ab^7}{18 = ab^3}$

$\left(\frac{389}{18}\right)^{\frac{1}{4}} = b^{4 \cdot \frac{1}{4}}$

$b = 2.156101859$

$y = ab^x$
$18 = a(2.156101859)^3$

$\frac{18}{10.02323291} = \frac{10.02323291\, a}{10.02323291}$

$1.795827217 = a$

$y = 2(2.16)^x$

less. 2

16. Engineers are draining a water reservoir until its depth is only 10 feet. The depth decreases exponentially as shown in the graph below. The engineers measure the depth after 1 hour to be 64 feet and after 4 hours to be 28 feet. Develop an exponential equation in $y = a(b)^x$ to predict the depth as a function of hours draining. Round a to the nearest integer and b to the nearest hundredth.

(1,64) (4,28)

$64 = ab^1$ $\quad 28 = ab^4$

$\frac{28 = ab^4}{64 = ab^1}$

$4.375^{\frac{1}{3}} = b^{3 \cdot \frac{1}{3}}$

$.759147243 = b$

$64 = a(.759147243)^1$

$84.30512077 = a$

$\boxed{y = 84(.76)^x}$

Common base

17. Solve each of the following for x:

a. $5^{3x} = 25^{x+1}$

$5^{3x} = 5^{2(x+1)}$

$3x = 2x + 2$

$x = 2$

b. $\left(\frac{1}{32}\right)^{x+3} = 16^{2x-4}$

$2^{-5(x+3)} = 2^{4(2x-4)}$

$-5x - 15 = 8x - 16$

$-13x = -1$

$\boxed{x = 1/13}$

c. $27^{3x+4} = 9^{2x-1}$

$3^{3(3x+4)} = 3^{2(2x-1)}$

$9x + 12 = 4x - 2$

$5x = -14$

$x = -14/5$

18. *Algebraically* determine the intersection point of the two exponential functions $y = 8^{7x-2}$ and $y = 16^{4x}$.

$8^{7x-2} = 16^{4x}$

$2^{3(7x-2)} = 2^{4(4x)}$

$21x - 6 = 16x$

$-6 = -5x$

$\frac{6}{5} = x$

$y = 602248.76$

19. *Algebraically* determine the x-intercepts of the exponential function $f(x) = 343^{2x+1} - \frac{1}{49}$.

$0 = 343^{2x+1} - \frac{1}{49}$

$\frac{1}{49} = 343^{2x+1}$

$7^{-2} = 7^{3(2x+1)}$

$-2 = 6x + 3$

$-5 = 6x$

$\boxed{\frac{-5}{6} = x}$

Growth: decay

20. For each of the following exponential functions below identify the initial value, tell if the function is increasing or decreasing, and tell the percent of increase or decrease.

a. $y = 624(1.03)^x$

initial 624

inc/dec. inc

% 3%

b. $B(t) = 98(0.97)^t$

initial 98

dec

dec. of 3%

c. $A(t) = 700(1.107)^x$

initial = 700

inc. 10.7%

21. The population of Munsonland grows by approximately 4.5% each year. If in 1995 there were 152 residents living in Munsonland, answer the following questions.

a. Write a formula, $P(t)$, that gives the population P at time t. $P(t) = 152(1.045)^t$

b. What is the population of Munsonland in the year 2006? $P(t) = 152(1.045)^{11} = 246.67$ 247

c. *Algebraically* determine, during what year the population of Munsonland will first reach 1000?

$\frac{1000}{152} = \frac{152(1.045)^t}{152}$

$\frac{125}{19} = 1.045^t$

$\frac{\log\left(\frac{125}{19}\right)}{\log(1.045)} = \frac{t \cdot \log(1.045)}{\log(1.045)}$

$t = 42.79$

during 2037 (2005+42.79)

22. Suppose that Miley has \$1200 to invest in one of three banks. Bank A offers an APR of 6.5%, compounded annually. Bank B offers an APR of 6% compounded monthly. Bank C charges a one-time fee to open an account of \$50 and offers an APR of 8% compounded quarterly. If Miley only plans to leave her money in the bank for 6 years, which bank should she choose?

A $1200(1+.065)^6$ \$1750.97

B $1200\left(1+\frac{.06}{12}\right)^{12\cdot 6}$ \$1718.45

C $1200\left(1+\frac{.08}{4}\right)^{4\cdot 6} - 50 = \$1880.12$

Bank C

23. Suppose you deposit \$400 in an account with an annual interest rate of 3% compounded quarterly. In how many years *to the nearest hundredth*, will you have 1000?

$1000 = 400\left(1+\frac{.03}{4}\right)^{4t}$

$\frac{1000}{400} = \frac{400}{400}(1.0075)^{4t}$

$2.5 = 1.0075^{4t}$

$\frac{\log(2.5)}{\log(1.0075)} = \frac{4t\log(1.0075)}{\log(1.0075)}$

$= 4t$

$t = 30.66$ years

24. Michelle deposited \$1750 into a money market account with an annual interest rate of 15% compounded monthly. In how many yearly, *to the nearest hundredth,* will Michelle have \$5000?

$\frac{5000}{1750} = \frac{1750}{1750}\left(1+\frac{.15}{12}\right)^{12t}$

$2.857142857 = (1.0125)^{12t}$

$\log(2.857142857) = 12t\log(1.0125)$

$7.04 = t$

25. The amount of money, E, in billions spent on health care expenditures can be estimated using the function $E(t) = 78.16(1.11)^t$, where t is time in years since 1970 (U.S. Census Bureau).

a. What were the health care expenditures in 1970? 78.16 billion

b. Is the cost increasing or decreasing? inc

c. By what percent is the cost increasing or decreasing? 11%

d. What are the expected health care expenditures in 2010? 40 years

2010 - 1970 = 40

$78.16(1.11)^{40}$

5080.4677

\$5080.47

26. Daisy decides to invest her money in her friend Ira's company. Daisy money can be modeled by the equation $D(t) = 7000(0.87)^t$.

a. How much did Daisy invest originally? 7000

b. Is her money increasing or decreasing? dec

c. By what percent is her money increasing or decreasing? 13%

d. In how many years, *to the nearest hundredth*, will Daisy only have \$1000?

$\frac{1000}{7000} = \frac{7000}{7000}(.87)^t$

$\log\left(\frac{1}{7}\right) = t\log(.87)$

$= \boxed{13.97}$ years

. Radioactive Strontium-90 has a half-life of 28 years. (Half-life is the amount of time needed for half of the substance to decay.) Find the percent, to the *nearest tenth*, of the substance would still be radioactive after:

a. 1 year. b. 7 years c. 42 years

$(.5)^{1/28}$

$.97554$ or

whats left. 97.55% left.

$(.5)^{7/28} = .840896$

84.09%

$(.5)^{42/28} = .35355$

or 35.4%

28. If a population was growing at a constant rate of 32% every 7 years, then what is its percent growth rate over at 2 year time span? Round to the *nearest tenth* of a percent.

$(1.040458)^2$ $(1.32)^{2/7} = 1.08255$ 8.3%

29. Evaluate each of the following:

a. $\log_3 9$

2

b. $\log_7 343$

3

c. $\log_2 \frac{1}{32}$

-5

d. $\log_{36} 6$

$1/2$

e. $\log_8 16$

$8^x = 16$

$2^{3x} = 2^4$ $3x = 4$

$x = 4/3$

f. $\log_{16} \frac{1}{64}$

$16^x = 64^{-1}$

$4^{2x} = 4^{-3}$

$2x = -3$

$x = -3/2$

g. $\log_{\frac{1}{9}} 27$

$3^{-2x} = 3^3$

$x = -3/2$

h. $\log_4 32$

$4^x = 32$

$2^{2x} = 2^5$

$x = 5/2$

30. Solve each of the following:

a. $\log_3 x = 5$

$3^5 = x$

$x = 243$

b. $\log_x 625 = \frac{4}{3}$

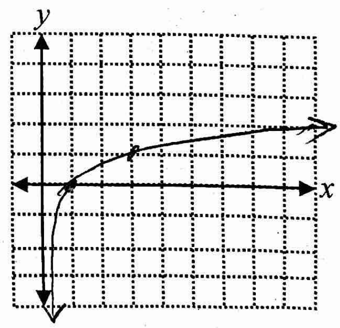
$x^{\frac{4}{3}} = 625^{3/4}$

$x = 125$

c. $\log_{27} x = \frac{2}{3}$

$27^{\frac{2}{3}} = x$

$x = 9$



31. a. On the accompanying graph grid, graph $y = \log_3 x$.

b. What is the domain and range of $y = \log_3 x$?

d: $x > 0$

R: $\mathbb{R}$

c. What is the equation of the asymptote?

$x = 0$ vertical.

32. Find the domain for each of the following:

a. $y = \log_5(7 - x)$

$7 - x > 0$

$-x > -7 \quad x < 7$

b. $y = \log_6(2x - 5) + 5x^2$

$2x - 5 > 0$

$2x > 5 \quad x > 5/2$

33. Solve each of the following for x: (Round your answers to the *nearest hundredth*)

a. $4^{\frac{x}{3}} = 12$

$\frac{x}{3}\log 4 = \log 12$

$\boxed{5.38}$

b. $14^{2x+1} = 283$

$2x+1 \cdot \log(14) = \log 283$

$\boxed{.57}$

c. $9(5)^{\frac{x}{8}-2} = 386$

$(5)^{\frac{x}{8}-2} = \frac{386}{9}$

$\frac{x}{8} - 2 \log 5 = \log\left(\frac{386}{9}\right)$

$\boxed{34.68}$

d. $6e^{5x} = 42$

$e^{5x} = 7$

$5x \cancel{\ln e} = \ln 7$

$.389 \quad$ or $\boxed{.39}$

34. Find the x and y intercepts of $y = 2e^{4x} - 20$, round to the *nearest hundredth* where applicable.

when $x = 0$

$y = 2e^{4(0)} - 20$

$y = -18$

$0 = 2e^{4x} - 20$

$\frac{20}{2} = \frac{2e^{4x}}{2}$

$10 = e^{4x}$

$\ln 10 = 4x \cancel{\ln e}$

$.575 \quad .58$

35. When a person has a cavity filled at the dentist, they usually get an injection of anesthesia which numbs their mouth for several hours. The amount of anesthesia still present in the tissue after t hours is given by the equation $A = 100e^{-0.5t}$.

a. How many ml did the dentist inject? 100

b. How much anesthesia is still present after 2 hours? $100e^{-.5(2)}$ $\boxed{\approx 36.79 \text{ ml}}$

c. When will the amount of anesthesia present be only 10 ml? Round to the *nearest hundredth.*

$10 = 100e^{-.5t}$

$.1 = e^{-.5t}$

$\ln .1 = -.5t \cancel{\ln e}$

$\boxed{4.61}$

36. An apple pie is taken out of the oven with an internal temperature of 325°F. It is placed on a rack in a room with a temperature of 72°F. After 10 minutes, the temperature of the pie is 200°F. (There was a question similar to this on the Newton's law of Cooling Class Worksheet.)

a. What will be the temperature of the pie 30 minutes after coming out of the oven?

$325 = ab^{0} + 72$

-72

$253 = a$

$200 = 253(b)^{10} + 72$

$-72 \qquad -72$

$128 = 253b^{10}$

10

$.9305928858$

$\boxed{.93}$

$253(.93)^{x} + 72 \qquad 253(.93)^{30} + 72$

$= 100.68°$

b. When will the temperature of the pie, to the *nearest tenth* of a minutes, be 80°F?

$80 = 253(.93)^{x} + 72$

-72

$8 = 253(.93)^{x}$

$\log \frac{8}{253} \div \log .93$

$x = 47.59$