NAME: ______________________

REVIEW FOR TEST ON UNITS #6 -#10
COMMON CORE ALGEBRA II

1. Factor completely each of the following: First consider the methods you have become very familiar with (GCF, Difference of Perfect Squares, Trinomials), and then consider grouping methods using "M substitutions" where helpful.

a. $3x^3-9x^2+12x$

$3x(x^2-3x+4)$

b. $12z^3-6z^2+18z$

$6z(2z^2-z+3)$

c. x^3-x^2+2x-2

$x^2(x-1)+2(x-1)$

$(x-1)(x^2+2)$

d. $6t^2-t-2$

$(3t-2)(2t+1)$

e. x^3-2x^2-x+2

$x^2(x-2)-1(x-2)$

$(x-2)(x^2-1)$

$(x-2)(x+1)(x-1)$

f. $6x^2-17x+10$

$(6x-5)(x-2)$

h. x^4+5x^2+6

$(x^2+3)(x^2+2)$

i. $20x^3+8x^2y-5xy^2-2y^3$

$4x^2(5x+2y)-y^2(5x+2y)$

$(5x+2y)(4x^2-y^2)$

$(5x+2y)(2x+y)(2x-y)$

$9x^5-a^2x^3-9x^2+a^2$

$x^3(9x^2-a^2)-1(9x^2-a^2)$

$(9x^2-a^2)(x^3-1)$

$(3x+a)(3x-a)(x^3-1)$

k. $2(2x+3)^2-72$

$2(2x+3+6)(2x+3-6)$

$2(2x+9)(2x-3)$

$2M^2-72$

$2(M^2-36)$

$2(M+6)(M-6)$

$6x^2+13xy-5y^2$

$(3x-1y)(2x+5y)$

$(3x-y)(2x+5y)$

m. x^4-81

$(x^2-9)(x^2+9)$

$(x+3)(x-3)(x^2+9)$

$(3-x)^2-(x+y)^2$

$((3-x)+(x+y))((3-x)-(x+y))$

$(3-x+x+y)(3-x-x-y)$

$(3+y)(3-2x-y)$

M^2-N^2

$(M-N)(M+N)$

o. $2y^5+4y^4-5y^3-10y^2-3y-6$

$2y^4(y+2)-5y^2(y+2)-3(y+2)$

$(y+2)(2y^4-5y^2-3)$

$(y+2)(2y^2+1)(y^2-3)$

2. Between what two consecutive integers does the largest root of $5x^2+2=-11x$ lie?

$5x^2+11x+2=0$

$x=\frac{-11\pm\sqrt{121-4(5)(2)}}{2(5)}$

$x=\frac{-11\pm\sqrt{81}}{10}=\frac{-11\pm 9}{10}$

$x=\frac{-11-9}{10}=-2$

$x=\frac{-11+9}{10}=\frac{-1}{5}$

The larger root is between -1 and 0.

3. Solve the following equations. Leave your answers in simplest radical form if needed.

a. $5x(x-3)+2=x^2+10x-4$

$5x^2-15x+2=x^2+10x-4$

$4x^2-25x+6=0$

$(4x-1)(x-6)=0$

$x=\frac{1}{4}\quad x=6$

$x=\{\frac{1}{4},6\}$

b. $-10r^2+46r+3=6r^2-18r+3$

$0=16r^2-64r$

$0=16r(r-4)$

$r=0\quad r=4$

$r=\{0,4\}$

c. $(x+3)^2-12=0$

$x^2+6x+9-12=0$

$x^2+6x-3=0$

$x=\frac{-6\pm\sqrt{36-4(1)(-3)}}{2(1)}$

$x=\frac{-6\pm\sqrt{48}}{2}$

$x=\frac{-6\pm 4\sqrt{3}}{2}=-3\pm 2\sqrt{3}$

d. $3x^3+2x^2-27x-18=0$

$x^2(3x+2)-9(3x+2)=0$

$(3x+2)(x^2-9)=0$

$(3x+2)(x+3)(x-3)=0$

$x=\frac{-2}{3}\quad x=-3\quad x=3$

$x=\{-3,-\frac{2}{3},3\}$

e. $(p-2)^2=4p$

$p^2-4p+4=4p$

$p^2-8p+4=0$

$p=\frac{8\pm\sqrt{64-4(1)(4)}}{2(1)}$

$p=\frac{8\pm\sqrt{48}}{2}$

$p=\frac{8\pm 4\sqrt{3}}{2}=4\pm 2\sqrt{3}$

f. $18x^3+9x^2-2x-1=0$

$9x^2(2x+1)-1(2x+1)=0$

$(2x+1)(9x^2-1)=0$

$(2x+1)(3x+1)(3x-1)=0$

$x=\frac{-1}{2}\quad x=\frac{-1}{3}\quad x=\frac{1}{3}$

$x=\{-\frac{1}{2},-\frac{1}{3},\frac{1}{3}\}$

4. Let $h(x)=x^2-6x$ and $g(x)=2x-7$. Find all values of x, for which $h(x)=g(x)$

a. *algebraically*

$h(x)=g(x)$

$x^2-6x=2x-7$

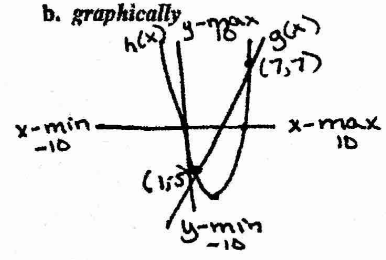
$x^2-8x+7=0$

$(x-7)(x-1)=0$

$x=7\quad x=1$

$x=\{1,7\}$

b. *graphically*



$x=\{1,7\}$

Sally tossed a ball in the air in such a way that the path of the ball was modeled by the equation $(t) = -2t^2 + 12t + 4$. In the equation, h represents the height of the ball in feet and t is the time in seconds. ll work must be done ***ALGEBRAICALLY*** (or NO credit will be given).

What was the initial height of the ball?

$h(0) = -2(0)^2 + 12(0) + 4 = 4$ feet

b. At what time is the ball at its highest point?

$t = \frac{-b}{2a} = \frac{-12}{2(-2)} = \frac{-12}{-4} = 3$ seconds

What is the maximum height of the ball?

$h(3) = -2(3)^2 + 12(3) + 4 = 22$ feet

After how many seconds did the ball reach 20 feet?

$20 = -2t^2 + 12t + 4$

$2t^2 - 12t + 16 = 0$

$2(t^2 - 6t + 8) = 0$

$2(t-4)(t-2) = 0$

$t = 4 \quad t = 2$

After 2 seconds

After how many seconds does the ball hit the ground? Round your answer to the *nearest hundredth*.

$0 = -2t^2 + 12t + 4$

$2t^2 - 12t - 4 = 0$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{12 \pm \sqrt{144 - 4(2)(-4)}}{2(2)} = \frac{12 \pm \sqrt{176}}{4}$

$t = \frac{12 + \sqrt{176}}{4} =$

$t = \frac{12 - \sqrt{176}}{4} =$

e each of the following quadratic inequalities. Express the solution set in interval notation.

$-10x + 4 > -3x(x-5) - 2$

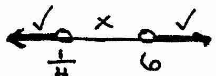
$-10x + 4 > -3x^2 + 15x - 2$

$^2 - 25x + 6 > 0$

$^2 - 25x + 6 = 0$

$-1)(x-6) = 0$

$= \frac{1}{4} \quad x = 6$



$(-\infty, \frac{1}{4}) \cup (6, \infty)$

$-1)^2 \geq -3x(x-2)$

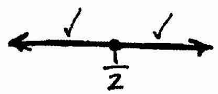
$-2x + 1 \geq -3x^2 + 6x$

$-4x + 1 \geq 0$

$^2 - 4x + 1 = 0$

$-1)(2x-1) = 0$

$= \frac{1}{2} \quad x = \frac{1}{2}$



$(-\infty, \infty)$

June 2003 until April 2004 JetBlue airlines stock (JBLU) was approximately worth $4t^2 + 80t - 360$ where P denotes the price of the stock in dollars and t corresponds to months, $= 1$ corresponding to January 2003. During what ***months*** was the stock value at least \$36.

$t^2 + 80t - 360 \geq 36$

$\geq 4t^2 - 80t + 396$

$= 4t^2 - 80t + 396$

$= 4(t^2 - 20t + 99)$

$= 4(t-11)(t-9)$

$t = 11 \quad t = 9$

During September 2003 and November 2003.

7. Place each of the quadratic functions in vertex form. Identify the turning point.

a. $g(x) = x^2 + 8x - 3$ $\quad (\frac{b}{2})^2 = (\frac{8}{2})^2 = 16$

$g(x) = x^2 + 8x + 16 - 16 - 3$

$g(x) = (x+4)^2 - 19$

vertex: $(-4, -19)$

b. $f(x) = 3x^2 - 12x + 13$ $\quad (\frac{b}{2})^2 = (\frac{-4}{2})^2 = 4$

$f(x) = 3(x^2 - 4x + \frac{13}{3})$

$f(x) = 3(x^2 - 4x + 4 - 4 + \frac{13}{3})$

$f(x) = 3[(x-2)^2 + \frac{1}{3}]$

$f(x) = 3(x-2)^2 + 1$ $\quad$ Vertex: $(2, 1)$

c. $f(x) = 2x^2 - 7x + 3$ $\quad (\frac{b}{2})^2 = (\frac{7}{2 \cdot 2})^2 = \frac{49}{16}$

$f(x) = 2(x^2 - \frac{7}{2}x + \frac{3}{2})$

$f(x) = 2(x^2 - \frac{7}{2}x + \frac{49}{16} - \frac{49}{16} + \frac{3}{2})$

$f(x) = 2[(x - \frac{7}{4})^2 - \frac{25}{16}]$

$f(x) = 2(x - \frac{7}{4})^2 - \frac{25}{8}$ $\quad$ vertex: $(\frac{7}{4}, \frac{-25}{8})$

d. $2p^2 + 8p = -3$ $\quad (\frac{b}{2})^2 = (\frac{4}{2})^2 = 4$

$2p^2 + 8p + 3 = y$

$2(p^2 + 4p + \frac{3}{2}) = y$

$2(p^2 + 4p + 4 - 4 + \frac{3}{2}) = y$

$2[(p+2)^2 - \frac{5}{2}] = y$

$y = 2(p+2)^2 - 5$

vertex: $(-2, -5)$

8. For each of the following equations of circles, determine both the circle's center and its radius. If its radius is not an integer, express it in decimal form rounded to the nearest *tenth*.

a. $(x-3)^2 + (y-8)^2 = 121$

$C(3, 8)$

$r = \sqrt{121} = 11$

b. $(x-5)^2 + (y+8)^2 = 4$

$C(5, -8)$

$r = \sqrt{4} = 2$

c. $x^2 + y^2 = 98$

$C(0, 0)$

$r = \sqrt{98} = 9.9$

d. $(x-3)^2 + (y+2)^2 = 50$

$C(3, -2)$

$r = \sqrt{50} = 7.1$

e. $x^2 + (y+3)^2 = 49$

$C(0, -3)$

$r = \sqrt{49} = 7$

f. $(x+7)^2 + y^2 = 225$

$C(-7, 0)$

$r = \sqrt{225} = 15$

9. Solve the following system of equations ***graphically and algebraically***:

$(x-3)^2 + (y-4)^2 = 9$

$y = x - 2$

Algebraically

$(x-3)^2 + ((x-2)-4)^2 = 9$

$(x-3)^2 + (x-6)^2 = 9$

$x^2 - 6x + 9 + x^2 - 12x + 36 = 9$

$2x^2 - 18x + 45 = 9$

$2x^2 - 18x + 36 = 0$

$2(x^2 - 9x + 18) = 0$

$2(x-6)(x-3) = 0$

$x = 6 \quad x = 3$

$x = 6$

$y = 6 - 2 = 4$

$x = 3$

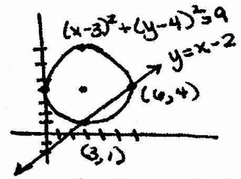
$y = 3 - 2 = 1$

$(6, 4)$ and $(3, 1)$

Graphically

$(x-3)^2 + (y-4)^2 = 9$

$C(3, 4) \quad r = 3$



$(6, 4)$ and $(3, 1)$

10. By completing the square on each of the quadratic expressions, determine the center and radius of a circle whose equation is shown below.

a. $x^2 + y^2 - 14x + 18y = -9$

$x^2 - 14x + y^2 + 18y = -9$

$(x^2 - 14x + 49) - 49 + (y^2 + 18y + 81) - 81 = -9$

$(x-7)^2 + (y+9)^2 - 130 = -9$

$(x-7)^2 + (y+9)^2 = 121$

$C(7, -9) \quad r = 11$

b. $x^2 - 6x + y^2 + 10y = 66$

$(x^2 - 6x + 9) - 9 + (y^2 + 10y + 25) - 25 = 66$

$(x-3)^2 + (y+5)^2 - 34 = 66$

$(x-3)^2 + (y+5)^2 = 100$

$C(3, -5) \quad r = 10$

11. Given a parabola whose directrix is $y=8$ and whose focus is $(7, 2)$, find the following

a. The vertex of the parabola

$(7,5)$

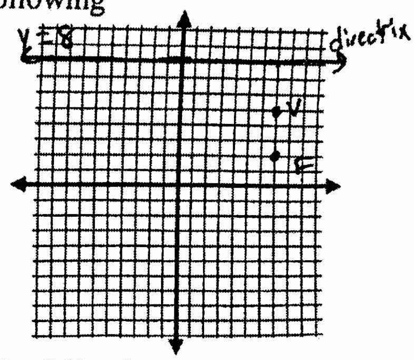
b. The equation of the parabola in ***vertex form.***

$p=-3 \qquad 4p=-12$

$(x-7)^2=-12(y-5)$

$-\frac{1}{12}(x-7)^2=y-5$

$-\frac{1}{12}(x-7)^2+5=y$



12. Given a parabola whose directrix is $y=-5$ and whose vertex is $(-6,-3)$, find the following

a. The focus of the parabola

$(-6,-1)$

b. The equation of the parabola in ***standard form.***

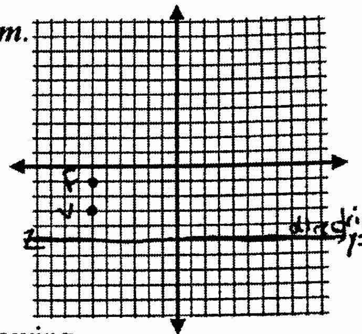
$p=2 \qquad 4p=8$

$(x+6)^2=8(y+3)$

$x^2+6x+36=8y+24$

$x^2+6x+12=8y$

$\frac{1}{8}x^2+\frac{3}{4}x+\frac{3}{2}=y$



13. Given a parabola whose focus is $(4, 7)$ and whose vertex is $(4, 3)$, find the following

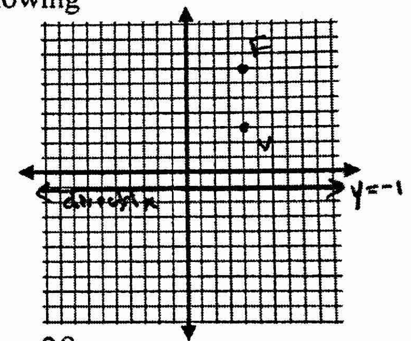
a. The directrix of the parabola

$y=-1$

b. The equation of the parabola

$p=4 \qquad 4p=16$

$(x-4)^2=16(y-3)$



14. Which equation represents a parabola with a focus of $(0, 4)$ and a directrix of $y=2$?

(1) $y=x^2+3$

(2) $y=-x^2+1$

(3) $y=\frac{x^2}{2}+3$

(4) $y=\frac{x^2}{4}+3$

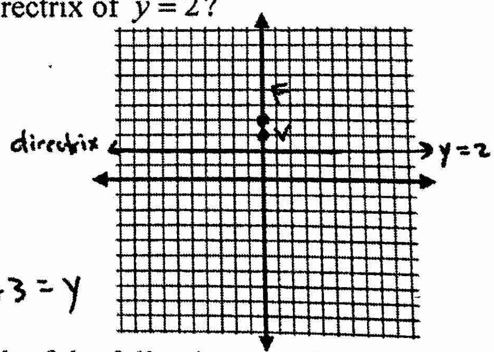
Vertex $(0,3)$

$p=1 \qquad 4p=4$

$(x-0)^2=4(y-3)$

$x^2=4y-12$

$x^2+12=4y \qquad \frac{x^2}{4}+3=y$



15. Write the equation of the function, and its domain, $y=\sqrt{x}$, after each of the following transformations:

a. shifted to the left 3 units

$y=\sqrt{x+3}$

b. shifted down 4 units

$y=\sqrt{x}-4$

c. shifted to the right 2 units and up 3 units

$y=\sqrt{x-2}+3$

d. a vertical stretch by a factor of 5 and shifted down 6 units. $y=5\sqrt{x}-6$

e. vertically compressed by a factor of 2 and shifted up 3 units.

$y=\frac{1}{2}\sqrt{x}+3$

f. horizontally stretch by a factor of 2

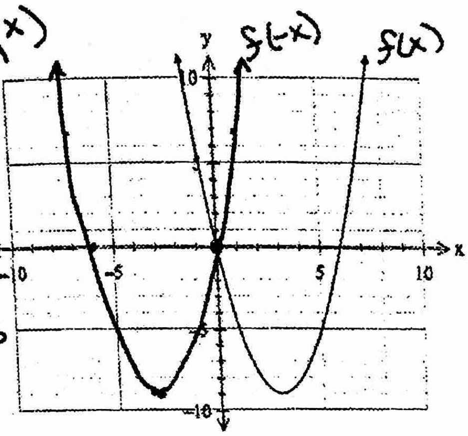
$y=\sqrt{\frac{1}{2}x}$

g. horizontally compressed by a factor of 3 and shifted down 4 units. $y=\sqrt{3x}-4$

16. The graph of $f(x)=x^2+6x$ is show below on two separate grids. Give an equation and sketch a graph for the functions (a) $f(-x)$ and (b) $-f(x)$.

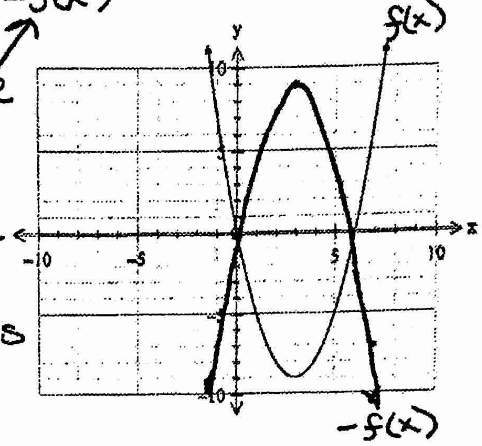
(a) $f(-x)$

negate x → reflect over y-axis



(b) $-f(x)$

negate y → reflect over x-axis



17. If $f(x)=-2x^2+5x-3$ and $g(x)$ is the reflection of $f(x)$ across the y-axis, write an equation of $g(x)$.

$g(x)=f(-x)=-2(-x)^2+5(-x)-3=-2x^2-5x-3$

18. If the point $(-3,-5)$ lies on the graph of a function $h(x)$ then what point *must* lie on the graph of the function $-h(x)$.

negate "y" $(^-3,5)$

19. The graph of $y=8-x^2$ represents the graph of $y=x^2$ after

(1) a vertical shift upwards of 8 units followed by a reflection in the x-axis.
(2) a reflection in the x-axis followed by a vertical shift of 8 units upward.
(3) a leftward shift of 8 units followed by a reflection in the y-axis.
(4) a reflection across the x-axis followed by a rightward shift of 8 units.

$y=x^2$ negate "y" reflect over x-axis
$y=-x^2$ shift up 8
same $\{ y=-x^2+8$, $y=8-x^2$

20. If the function $y=-f(x-4)+3$ were graphed, write the transformations to the graph of $y=f(x)$ in the order that they occur.

① Horizontal
① Stretch
② Reflect
③ Shift
② Vertical
① Stretch ② Reflect ③ Shift

$y=\underset{\#2}{-}f(x\underset{\#1}{-4})\underset{\#3}{+3}$

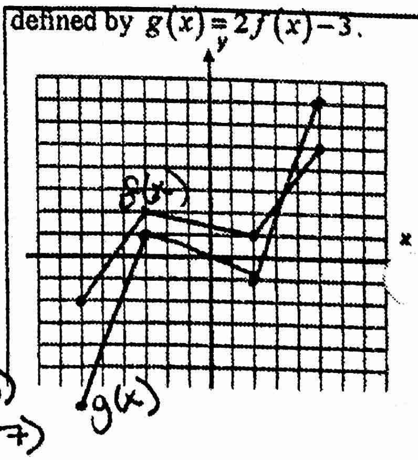
① shift to the right 4 units
② reflection in the x-axis
③ vertical shift up 3 units

21. The graph of $y=f(x)$ is shown below. Consider the function $y=g(x)$ defined by $g(x)=2f(x)-3$.

a. What two transformations have occurred to the graph of f in order to produce the graph of g? Specify both the transformations and their order.

$g(x)=\underset{\#1}{2}f(x)\underset{\#2}{-3}$

① Vertical stretch by a factor of 2.
② Vertical shift down 3 units



b. Graph and label $y=g(x)$

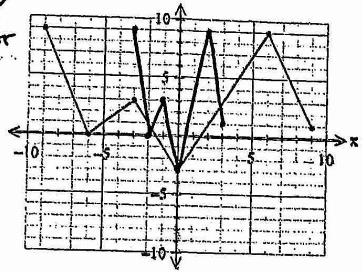
$g(x)=2"y"-3$
$=2(y)-3$

f(x) (-6,-2) (-3,2) (2,1) (5,5)
g(x) (-6,-7) (-3,1) (2,-1) (5,7)

22. The graph of $f(x)$ is shown on the grid below. Sketch a graph of $f(3x)$ on the same set of axes. divide x values by 3

State the domain of the two functions:

horizontally compressed by a factor of 3



Domain of $f(x)$:

[-9,9]

$\{x \mid -9 \le x \le 9\}$

Domain of $f(3x)$:

[-3,3]

$\{x \mid -3 \le x \le 3\}$

23. If $f(x)$ is an even function and $f(2)=-5$ then what is the value of $4f(2)+2f(-2)$?

opposite x's, same y's

(2,-5)

(-2,-5)

$4f(2)+2f(-2)$

$4(-5)+2(-5)$

$-20+(-10)=-30$

24. If $f(x)$ is an odd function and $f(3)=4$ then what is the value of $2f(3)+4f(-3)$

opposite x's, opposite y's

(3,4)

(-3,-4)

$2f(3)+4f(-3)$

$2(4)+4(-4)$

$8+(-16)=-8$

25. Algebraically determine if each of the following functions are even, odd or neither.

(a) $y=3x-3$ NEITHER

$y=3(-x)-3$

$y=-3x-3$

(b) $y=5x^2+12$ EVEN

$y=5(-x)^2+12$

$y=5x^2+12$

opposite x's, same y's

(c) $y=x^3+7x-3$ NEITHER

$y=(-x)^3+7(-x)-3$

$y=-x^3-7x-3$

(d) $f(x)=2x^7-6x^5$ ODD

$f(-x)=2(-x)^7-6(-x)^5$

$f(-x)=-2x^7+6x^5$

opposite x's, opposite y's

(e) $g(x)=-3x^6-7x^2$ EVEN

$g(-x)=-3(-x)^6-7(-x)^2$

$g(-x)=-3x^6-7x^2$

opposite x's, same y's

(f) $f(x)=\dfrac{2x^2}{3x+1}$ NEITHER

$f(-x)=\dfrac{2(-x)^2}{3(-x)+1}$

$f(-x)=\dfrac{2x^2}{-3x+1}$

(g) $y=\dfrac{2x}{x^2-1}$ ODD

$y=\dfrac{2(-x)}{x^2-1}$

$y=\dfrac{-2x}{x^2-1}$

opposite x's, opposite y's

(h) $f(x)=\dfrac{x^3}{3x-6}$ NEITHER

$f(-x)=\dfrac{(-x)^3}{3(-x)-6}$

$f(-x)=\dfrac{-x^3}{-3x-6}$

26. State the domain of each function.

a. $h(x)=\sqrt{2x-8}$

$2x-8\ge 0$

$2x\ge 8$

$x\ge 4$

$\{x \mid x\ge 4\}$

$[4,\infty)$

b. $g(x)=\dfrac{3x}{\sqrt{3-x}}$

$3-x>0$

$-x>-3$

$x<3$

$\{x \mid x<3\}$

$(-\infty,3)$

c. $f(x)=\dfrac{2x-6}{\sqrt{x^2-4x-12}}$

$x^2-4x-12>0$

$x^2-4x-12=0$

$(x-6)(x+2)=0$

$x=6 \quad x=-2$

✓ ✗ ✓

-2 6

$\{x \mid x<-2 \text{ or } x>6\}$

$(-\infty,-2)\cup(6,\infty)$

d. $f(x)=\sqrt{3x+12}$

$3x+12\ge 0$

$3x\ge -12$

$x\ge -4$

$\{x \mid x\ge -4\}$

$[-4,\infty)$

e. $g(x)=\dfrac{x}{x+7}$

$x+7\ne 0$

$x\ne -7$

f. $t(x)=\dfrac{x-2}{3x^2-20x-7}$

$3x^2-20x-7\ne 0$

$(3x+1)(x-7)\ne 0$

$x\ne -\frac{1}{3} \quad x\ne 7$

g. $g(x)=\dfrac{3x^2-7x-4}{2}$

All real numbers

h. $f(x)=|x+8|$

All real numbers

i. $v(x)=\dfrac{2x-4}{\sqrt{x^2-9}}$

$x^2-9>0$

$x^2-9=0$

$(x+3)(x-3)=0$

$x=-3 \quad x=3$

✓ ✗ ✓

-3 3

$\{x \mid x<-3 \text{ or } x>3\}$

$(-\infty,-3)\cup(3,\infty)$

j. $f(x)=\sqrt{3x^2+14x-5}$

$3x^2+14x-5\ge 0$

$3x^2+14x-5=0$

$(3x-1)(x+5)=0$

$x=\frac{1}{3} \quad x=-5$

✓ ✗ ✓

-5 1/3

$\{x \mid x\le -5 \text{ or } x\ge 1/3\}$

$(-\infty,-5]\cup[1/3,\infty)$

k. $\sqrt{8x-2x^2}$

$8x-2x^2\ge 0$

$8x-2x^2=0$

$0=2x^2-8x$

$0=2x(x-4)$

$x=0 \quad x=4$

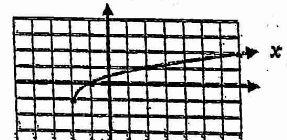
✗ ✓ ✗

0 4

$\{x \mid 0\le x\le 4\}$

$[0,4]$

l.



$[-2,\infty)$

$\{x \mid x\ge -2\}$

27. Expressed with fractional exponents, $\dfrac{2}{\sqrt[5]{x^3}}$ is equivalent to

27. (2)

(1) $2x^{-\frac{5}{3}}$ (2) $2x^{-\frac{3}{5}}$ (3) $\frac{1}{2}x^{-\frac{5}{3}}$ (4) $\frac{1}{2}x^{-\frac{3}{5}}$

28. Place each of the following power functions in the form $y=ax^b$.

a. $y=5\sqrt{x}$

$y=5x^{1/2}$

b. $y=\frac{1}{2x^3}$

$y=\frac{1}{2}x^{-3}$

c. $y=\sqrt{4x^3}$

$y=2x^{3/2}$

d. $y=\frac{3}{2\sqrt[3]{x}}$

$y=\frac{3}{2}x^{-1/3}$

29. Simplify each of the following:

a. $\sqrt[3]{-54a^6b^7}$

$\sqrt[3]{-27a^6b^6}\ \sqrt[3]{2b}$

$-3a^2b^2\ \sqrt[3]{2b}$

b. $\sqrt[4]{32x^{12}y^{14}z^9}$

$\sqrt[4]{16x^{12}y^{12}z^8}\ \sqrt[4]{2y^2z}$

$2x^3y^3z^2\ \sqrt[4]{2y^2z}$

c. $\sqrt[3]{-125x^5y^4}$

$\sqrt[3]{-125x^3y^3}\ \sqrt[3]{x^2y}$

$-5xy\ \sqrt[3]{x^2y}$

30. Solve for x:

a. $5+3\sqrt{x-4}=26$

$3\sqrt{x-4}=21$

$\sqrt{x-4}=7$

$x-4=49$

$x=53$

b. $-3\sqrt{2x-5}+8=-1$

$-3\sqrt{2x-5}=-9$

$\sqrt{2x-5}=3$

$2x-5=9$

$2x=14$

$x=7$

c. $2\sqrt{x-7}=x-10$

$\sqrt{x-7}=\frac{1}{2}x-5$

$x-7=(\frac{1}{2}x-5)^2$

$x-7=\frac{1}{4}x^2-5x+25$

$0=\frac{1}{4}x^2-6x+32$

$0=x^2-24x+128$

$0=(x-16)(x-8)$

$x=16$ $\cancel{x=8}$ reject

d. $\sqrt{x+8}-1=2x$

$\sqrt{x+8}=2x+1$

$x+8=(2x+1)^2$

$x+8=4x^2+4x+1$

$0=4x^2+3x-7$

$0=(4x+7)(x-1)$

$\cancel{x=\frac{-7}{4}}$ reject $x=1$

31. Solve each of the following quadratics using the quadratic formula leave your answer in simplest radical form.

a. $5x^2=12-28x$

$5x^2+28x-12=0$

$x=\frac{-28\pm\sqrt{784-4(5)(-12)}}{2(5)}$

$x=\frac{-28\pm\sqrt{1024}}{10}$

$x=\frac{-28\pm 32}{10}$

$x=\{-\frac{1}{5}, 6\}$

b. $x^2+8x+4=0$

$x=\frac{-8\pm\sqrt{64-4(1)(4)}}{2(1)}$

$x=\frac{-8\pm\sqrt{48}}{2}$

$x=\frac{-8\pm4\sqrt{3}}{2}$

$x=-4\pm2\sqrt{3}$

c. $x^2=10x-74$

$x^2-10x+74=0$

$x=\frac{10\pm\sqrt{100-4(1)(74)}}{2(1)}$

$x=\frac{10\pm\sqrt{-196}}{2}$

$x=\frac{10\pm14i}{2}$

$x=5\pm7i$

32. A projectile is fired from a height of 4.5 meters above the ground of a level surface at an initial upwards velocity of 30 meters per second. Its height above level ground is given by the equation $h=-4.9t^2+30t+4.5$. After how many seconds, t, will the ball hit the ground. Find using the quadratic formula and round to the nearest tenth of a second.

$t=\frac{-30\pm\sqrt{900-4(-4.9)(4.5)}}{2(-4.9)}$

$t=\frac{-30\pm\sqrt{988.2}}{-9.8}$

$t=\frac{-30+\sqrt{988.2}}{-9.8}=\cancel{-.1}$ reject

$t=\frac{-30-\sqrt{988.2}}{-9.8}=6.3$

The ball hits the ground after 6.3 seconds

33. Simplify each of the following: $i^0=1$ $i^1=i$ $i^2=-1$ $i^3=-i$ $i^n=i^{\text{remainder of } \frac{n}{4}}$

(a) i^2

-1

(b) i^{37}

i^1

i

(c) i^{56}

i^0

1

(d) i^{78}

i^2

-1

(i) $\sqrt{-36}$

$\sqrt{-1}\sqrt{36}$

$6i$

(j) $-8\sqrt{-25}$

$-8\sqrt{-1}\sqrt{25}$

$-8\cdot5\cdot i$

$-40i$

(k) $-6\sqrt{-8}$

$-6\sqrt{-1}\sqrt{8}$

$-6i\sqrt{4}\sqrt{2}$

$-6i\cdot2\sqrt{2}$

$-12i\sqrt{2}$

(l) $2\sqrt{-72}$

$2\sqrt{-1}\sqrt{72}$

$2i\sqrt{36}\sqrt{2}$

$2i\cdot6\sqrt{2}$

$12i\sqrt{2}$

34. Perform the indicated operations below and leave each answer in simplest $a+bi$ form

(a) $(2+7i)+(8-5i)$

$2+7i+8-5i$

$10+2i$

(b) $(12+5i)-(9-3i)$

$12+5i-9+3i$

$3+8i$

(c) $7i\cdot3i$

$21i^2$

$21(-1)$

-21

(d) $4i(8-10i)$

$32i-40i^2$

$32i-40(-1)$

$40+32i$

(e) $(7-3i)(6-5i) = 42-35i-18i+15i^2 = 42-53i-15 = 27-53i$

35. Solve the equation $x^2-4x+5=0$ over the set of complex numbers. Put your answers in simplest $a+bi$ form.

$x=\frac{4\pm\sqrt{16-4(1)(5)}}{2(1)}$

$x=\frac{4\pm\sqrt{-4}}{2}$

$x=\frac{4\pm2i}{2}$

$x=2\pm i$

Summary of the Discriminant

Discriminant	Roots of Equation	x-Intercepts
1. $b^2-4ac>0$ and perfect square	Two unequal real, rational roots.	Two rational x-intercepts
2. $b^2-4ac=0$	One real, double root.	One rational x-intercept. (tangent to x-axis)
3. $b^2-4ac>0$ and not a perfect square	Two unequal real, irrational roots.	Two irrational x-intercepts
4. $b^2-4ac<0$	Two imaginary roots.	No x-intercept

36. Select the choice below that describes the graph of the given parabolas below:
(1) It is tangent to the x-axis (3) It intersects the x-axis at 2 points
(2) It lies entirely above the x-axis (4) It lies entirely below the x-axis

a. $y=2x^2+7x-4$
b^2-4ac
$49-4(2)(-4)$
81
(3)

b. $y=x^2-8x+25$
b^2-4ac
$64-4(1)(25)$
-36
(2)

c. $y=4x^2+4x+1$
b^2-4ac
$16-4(4)(1)$
0
(1)

d. $y=2x^2+6x-15$
b^2-4ac
$36-4(2)(-15)$
156
(3)

37. Select the choice below that describes the nature of the roots of the given equations below:
(1) real, rational, and unequal (3) real, rational, and equal
(2) real, irrational, and unequal (4) imaginary

a. $4x^2=4x+7$
$4x^2-4x-7=0$
b^2-4ac
$16-4(4)(-7)$
128
(2)

b. $8x(2x-1)=-1$
$16x^2-8x+1=0$
b^2-4ac
$64-4(16)(1)$
0
(3)

c. $x-8=x^2-2x$
$0=x^2-3x+8$
b^2-4ac
$9-4(1)(8)$
-23
(4)

d. $8x^2-7x-3=5(x^2-1)$
$8x^2-7x-3=5x^2-5$
$3x^2-7x+2=0$
b^2-4ac
$49-4(3)(2)$
25
(1)

38. Find all values of k such that the equation $3x^2-2x+k=0$ has imaginary roots. $b^2-4ac<0$
$b^2-4ac<0$
$4-4(3)(k)<0$
$4-12k<0$
$-12k<-4$ → $k>\frac{1}{3}$

39. Find the largest integer value of k that makes the graph of $y=kx^2-8x+2$ cross the x-axis twice. $b^2-4ac>0$
$b^2-4ac>0$
$64-4(k)(2)>0$
$64-8k>0$
$-8k>-64$ → $k<8$
The largest integer value is $k=7$.

40. Find the smallest integer value of k such that the equation $y=kx^2-2x+5$ has imaginary roots. $b^2-4ac<0$
$b^2-4ac<0$
$4-4(k)(5)<0$
$4-20k<0$
$-20k<-4$
$k>\frac{1}{5}$
The smallest integer value is $k=1$.

41. Find the value(s) of k that make the graph of $y=kx^2+10x+1$ tangent to the x-axis. $b^2-4ac=0$
$b^2-4ac=0$
$100-4(k)(1)=0$
$100-4k=0$
$100=4k$
$25=k$

42. The roots of a quadratic equation may be: 42. (1)
(1) rational and equal (2) irrational and equal (3) imaginary and equal (4) none of these
$b^2-4ac=0$

43. Which of the following could not be the solution of a quadratic equation? 43. (3)
(1) {-3} (2) {2, -3} (3) $\{2-3i, -2-3i\}$ (4) $\{3i, -3i\}$

44. Which equation has imaginary solutions? $b^2-4ac<0$ 44. (4)
(1) $x^2-7=0$ (2) $x^2-2x+1=0$ (3) $x^2-2x-1=0$ (4) $x^2-2x+2=0$
b^2-4ac | $0-4(1)(-7)=28$
b^2-4ac | $4-4(1)(1)=0$
b^2-4ac | $4-4(1)(-1)=8$
b^2-4ac | $4-4(1)(2)=-4$

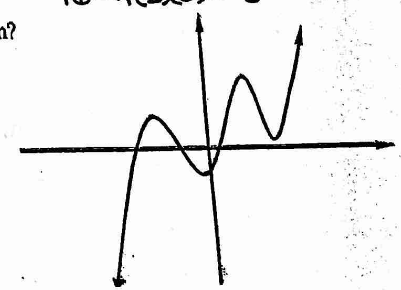
45. Which equation has roots that are real, rational, and equal? $b^2-4ac=0$ 45. (2)
(1) $x^2-\frac{1}{16}=0$ (2) $4x^2-4x+1=0$ (3) $x^2+x+1=0$ (4) $x^2-12x-36=0$
$b^2-4ac=0$
$0-4(1)(\frac{1}{16})=-\frac{1}{4}$
b^2-4ac
$16-4(4)(1)=0$

46. If the roots of the equation $2x^2-3x+c=0$ are real and irrational, the value of c may be → $b^2-4ac>0$ not a perfect square 46. (4)
(1) 1 (2) 2 (3) 0 (4) -1
b^2-4ac | $9-4(2)(1)$ | 1
b^2-4ac | $9-4(2)(2)$ | -7
b^2-4ac | $9-4(2)(0)$ | 9
b^2-4ac | $9-4(2)(-1)$ | 17

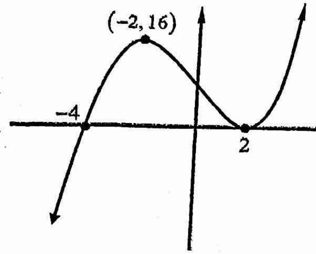
47. The roots of the equation $ax^2+4x=-2$ are real and equal if a has a value of $b^2-4ac=0$ 47. (2)
$2x^2+4x+2=0$
(1) 1 (2) 2 (3) 3 (4) 4
b^2-4ac
$16-4(1)(2)$
8
b^2-4ac
$16-4(2)(2)$
0

48. Which value of k will make the roots of $2x^2-4x+k=0$ imaginary? $b^2-4ac<0$ 48. (3)
(1) -2 (2) 2 (3) 3 (4) -3
$16-4(2)(-2)=32$
$16-4(2)(2)=0$
$16-4(2)(3)=-8$

49. Could the following graph be that of a cubic function?
Explain your answer.
No! A cubic can have a maximum of 3 zeroes and 2 turning points.



50. The cubic polynomial shown below crosses the x-axis at $x=-4$ and is tangent to the x-axis at $x=2$. It has a turning point at $(-2,16)$. Find the equation of the cubic function below in standard form.



$y=a(x+4)(x-2)^2$

$16=a(-2+4)(-2-2)^2$

$16=a(2)(16)$

$16=32a$

$\frac{1}{2}=a$

$y=\frac{1}{2}(x+4)(x-2)^2$

$y=\frac{1}{2}(x+4)(x^2-4x+4)$

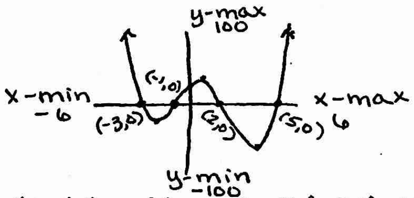
$y=\frac{1}{2}(x^3-4x^2+4x+4x^2-16x+16)$

$y=\frac{1}{2}(x^3-12x+16)$

$y=\frac{1}{2}x^3-6x+8$

51. What are the solutions of the equation $x^4+19x+30=3x^3+15x^2$?

$x^4-3x^3-15x^2+19x+30=0$



$x=\{-3,-1,2,5\}$

52. What are the solutions of the equation $(3x^2+7x)^2+6(3x^2+7x)+8=0$?

$M^2+6M+8=0$

$(M+2)(M+4)=0$

$(3x^2+7x+2)(3x^2+7x+4)=0$

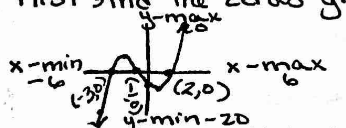
$(3x+1)(x+2)(3x+4)(x+1)=0$

$x=-\frac{1}{3}$ $x=-2$ $x=\frac{-4}{3}$ $x=-1$

$x=\{-2,-\frac{4}{3},-1,\frac{-1}{3}\}$

53. What are the factors of the polynomial expression x^3+2x^2-5x-6?

First find the zeroes graphically then write it in factored form.



$x=\{-3,-1,2\}$

x^3+2x^2-5x-6

$(x+3)(x+1)(x-2)$

54. Find the equation of the cubic function of the form $y=x^3+bx^2+cx+d$ if its x intercepts are {-1, 3, 5}.

$*a=1*$

$y=a(x+1)(x-3)(x-5)$

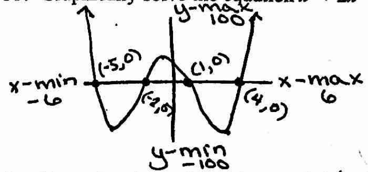
$y=(x+1)(x-3)(x-5)$

$y=(x+1)(x^2-8x+15)$

$y=x^3-8x^2+15x+x^2-8x+15$

$y=x^3-7x^2+7x+15$

55. Graphically solve the equation $x^4+2x^3-21x^2-22x+40=0$.



$x=\{-5,-2,1,4\}$

56. Given that $(2x+1)$ is a factor of $6x^3+29x^2-7x-10$, find the other two factors algebraically.

$$\begin{array}{r} 3x^2+13x-10 \\ 2x+1\,\overline{)\,6x^3+29x^2-7x-10} \\ -(6x^3+3x^2) \\ \hline 26x^2-7x \\ -(26x^2+13x) \\ \hline -20x-10 \\ -(-20x-10) \\ \hline 0 \end{array}$$

$6x^3+29x^2-7x-10$

$(2x+1)(3x^2+13x-10)$

$(2x+1)(3x-2)(x+5)$

57. Write the following rational function in its quotient-remainder form:

$y=\frac{4x^3+2x^2-4x+1}{2x+7}$

$$\begin{array}{r} 2x^2-6x+19 \\ 2x+7\,\overline{)\,4x^3+2x^2-4x+1} \\ -(4x^3+14x^2) \\ \hline -12x^2-4x \\ -(-12x^2-42x) \\ \hline 38x+1 \\ -(38x+133) \\ \hline -132 \end{array}$$

$y=\frac{4x^3+2x^2-4x+1}{2x+7}=2x^2-6x+19-\frac{132}{2x+7}$

58. Find the inverse of each of the following functions.

(a) $y=\frac{x}{x+2}$

$x=\frac{y}{y+2}$

$xy+2x=y$

$2x=y-xy$

$2x=y(1-x)$

$\frac{2x}{1-x}=y$

(b) $f(x)=\frac{3x+1}{2x-4}$

$x=\frac{3y+1}{2y-4}$

$2xy-4x=3y+1$

$2xy-3y=4x+1$

$y(2x-3)=4x+1$

$y=\frac{4x+1}{2x-3}$

(c) $g(x)=\frac{5-3x}{x+7}$

$x=\frac{5-3y}{y+7}$

$xy+7x=5-3y$

$xy+3y=5-7x$

$y(x+3)=5-7x$

$y=\frac{5-7x}{x+3}$

… 2x … following fractional equations. Be sure to check for extraneous roots.

a. $x+\frac{5}{2}=\frac{6}{x}$ $\qquad$ * $x \neq 0$ *

$2x^2+5x=12$

$2x^2+5x-12=0$

$(2x-3)(x+4)=0$

$x=\frac{3}{2} \quad x=-4$

$x=\{-4,\frac{3}{2}\}$

LCD: $30(x+2)$

b. $\frac{x-1}{x+2}-\frac{5}{6}=\frac{x-10}{15}$ $\qquad$ * $x \neq -2$ *

$30(x+2)\left[\frac{x-1}{x+2}-\frac{5}{6}=\frac{x-10}{15}\right]$

$30(x-1)-5(5(x+2))=2(x+2)(x-10)$

$30x-30-25x-50=2(x^2-8x-20)$

$5x-80=2x^2-16x-40$

$0=2x^2-21x+40$

$0=(2x-5)(x-8)$

$x=\frac{5}{2} \quad x=8$

LCD: $x(x-7)$

c. $\frac{x-3}{x-7}-\frac{1}{x}=\frac{28}{x^2-7x}$ $\qquad$ * $x \neq \{0,7\}$ *

$x(x-7)$

$x(x-7)\left[\frac{x-3}{x-7}-\frac{1}{x}=\frac{28}{x(x-7)}\right]$

$x(x-3)-1(x-7)=28$

$x^2-3x-x+7=28$

$x^2-4x-21=0$

$(x-7)(x+3)=0$

$\cancel{x=7}$ $\quad x=-3$

reject

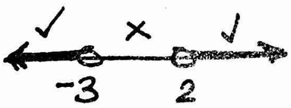
60. Solve each of the following rational inequalities. Use any appropriate notation and graph your solution on a number line.

a. $\frac{x-2}{x+3}>0$ $\qquad$ * $x \neq -3$ *

$x+3\left[\frac{x-2}{x+3}=0\right]$

$x-2=0$

$x=2$



$\{x|x<-3 \text{ or } x>2\}$

$(-\infty,-3)\cup(2,\infty)$

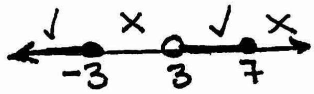
b. $\frac{x^2-4x-21}{x-3}\le 0$ $\qquad$ * $x \neq 3$ *

$x-3\left[\frac{x^2-4x-21}{x-3}=0\right]$

$x^2-4x-21=0$

$(x-7)(x+3)=0$

$x=7 \quad x=-3$



$\{x|x\le -3 \text{ or } 3<x\le 7\}$

$(-\infty,-3]\cup(3,7]$

c. $\frac{x^2+2x-3}{x^2-7x+6}>0$

* $x^2-7x+6 \neq 0$

$(x-6)(x-1)\neq 0$

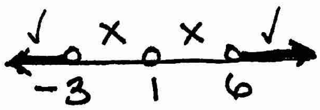
$x \neq \{1,6\}$

$\frac{x^2+2x-3}{x^2-7x+6}=0$

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=-3 \quad x=1$



$\{x|x<-3 \text{ or } x>6\}$

$(-\infty,-3)\cup(6,\infty)$

1. Determine the power function in each polynomial that describes the polynomial's long-run behavior.

a. $y=6x^4+5x^3-2x^2+x-7$ $\qquad$ b. $y=-5x^4+7x^3-8x^5+3x+2$ $\qquad$ c. $y=7x^3+2x^4-9x^5$

$y=6x^4$ $\qquad$ $y=-8x^5$ $\qquad$ $y=-9x^5$