Hilton High School

Algebra II (Common Core)

Practice Test 4

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1. A doctor wants to test the effectiveness of a new drug on her patients. She separates her sample of patient into two groups and administers the drug to only one of these groups. She then compares the results. Which type of study *best* describes this situation?

   (1) Census

   (2) Survey

   (3) Observation

   (4) Controlled experiment

2. If $f(x) = \frac{x}{x^2-16}$. What is the value of $f(-10)$?

   (1) $-\frac{5}{2}$

   (2) $-\frac{5}{42}$

   (3) $\frac{5}{58}$

   (4) $\frac{5}{18}$

3. An auditorium has 21 rows of seats. The first row has 18 seats, and each succeeding row has two more seats than the previous row. How many seats are in the auditorium?

   $\sum_{n=1}^{21} 18 + (n-1)(2)$

   (1) 540

   (2) 567

   (3) 760

   (4) 798

4. Expressed as a function of a positive acute angle, $\cos(-305°)$ is equal to

   (1) $-\cos 55°$

   (2) $\cos 55°$

   (3) $-\sin 55°$

   (4) $\sin 55°$

5. The value of x in the equation $4^{2x+5} = 8^{3x}$ is

(1) 1

(2) 2

(3) 5

(4) -10

Use this space for computations.

$2(2x + 5) = 3(3x)$
$4x + 10 = 9x$
$10 = 5x$
$2 = x$

6. What is the value of x in the equation $\log_5 x = 4$?

(1) 1.16

(2) 20

(3) 625

(4) $1{,}024$

7. The expression $\sqrt[4]{16x^2y^7}$ is equivalent to

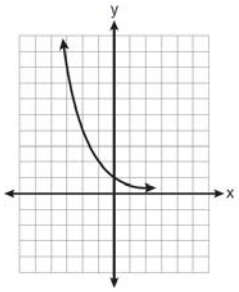
(1) $2x^{\frac{1}{2}}y^{\frac{7}{4}}$

(2) $2x^8y^{28}$

(3) $4x^{\frac{1}{2}}y^{\frac{7}{4}}$

(4) $4x^8y^{28}$

8. Which equation is represented by the graph below?



(1) $y = 5^x$

(2) $y = 0.5^x$

(3) $y = 5^{-x}$

(4) $y = 0.5^{-x}$

9. What is the fifteenth term of the geometric sequence $-\sqrt{5}, \sqrt{10}, -2\sqrt{5}, \ldots$?

(1) $-128\sqrt{5}$

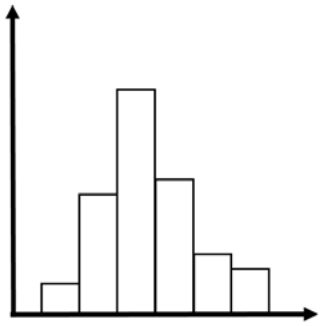
(2) $128\sqrt{10}$

(3) $-16384\sqrt{5}$

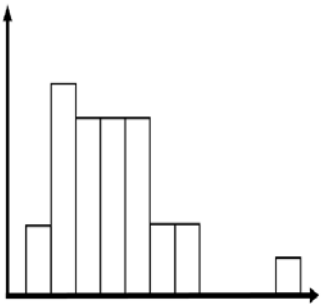
(4) $16384\sqrt{10}$

10. Which histogram depicts data that is *skewed left*?

Use this space for computations.



(1)



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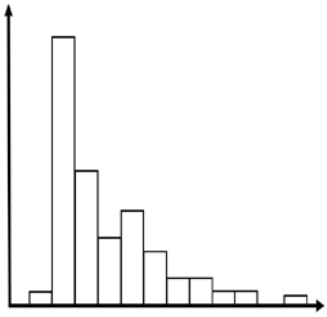

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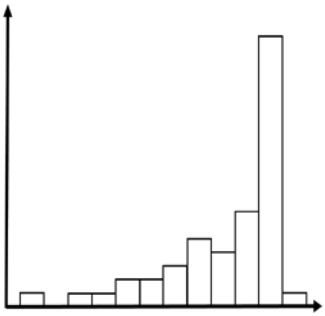


(4)

11. What is the period of the function $f(\theta) = -2\cos 3\theta$?

(1) π

(2) $\frac{2\pi}{3}$

(3) $\frac{3\pi}{2}$

(4) 2π

12. What is the range of $f(x) = (x+4)^2 + 7$?

(1) $y \geq -4$

(2) $y \geq 4$

(3) $y = 7$

(4) $y \geq 7$

13. The table below shows the results of a survey in which workers between the ages of 26 and 45 were asked if they have at least one month's income set aside for emergencies.

Use this space for computations.

	Men	Women
Less than one month's income	66	83
One month's income or more	76	62

If two *different* workers are randomly selected, find the probability that they both have one month's income or more set aside for emergencies.

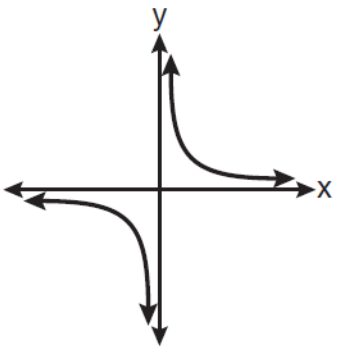
(1) 64%

(2) 24%

(3) 48%

(4) 23%

14. Which graph represents a relation that is *not* a function?



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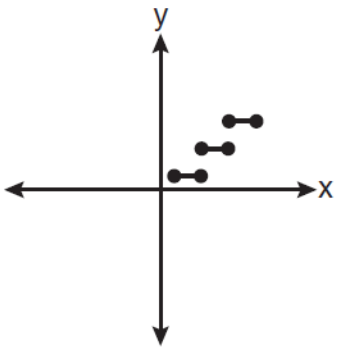

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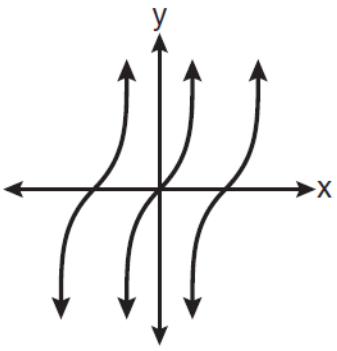

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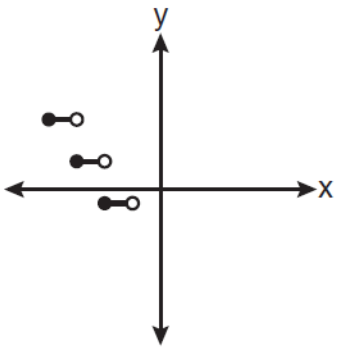


(4)

15. The value of $\tan 126°43'$ to the *nearest ten-thousandth* is

Use this space for computations.

(1) -1.3407

(2) -1.3408

(3) -1.3548

(4) -1.3549

16. The expression $\frac{3-\sqrt{8}}{\sqrt{3}}$ is equivalent to

(1) $\frac{\sqrt{3}-2\sqrt{6}}{\sqrt{3}}$

(2) $-\sqrt{3}+\frac{2}{3}\sqrt{6}$

(3) $\frac{3-\sqrt{24}}{3}$

(4) $\sqrt{3}-\frac{2}{3}\sqrt{6}$

17. Jenna invests \$25,000 in an account that pays 4.75% annual interest compounded continuously. Using the formula $A = Pe^{rt}$, where A is the amount in the account after t years, P is the principal invested, and r is the annual interest rate, how many years, to the *nearest tenth*, will it take for Jenna's investment to triple?

$75{,}000 = 25{,}000e^{0.0475t}$

$3 = e^{0.0475t}$

$\ln(3) = 0.0475t$

(1) 10.0

(2) 14.6

(3) 23.1

(4) 24.0

18. Professor Robert E. Kelly kept his cool during an interview about South Korean politics when his two children interrupted him, live on air, on BBC World News. The video went viral! On the first day, the video was viewed 70,000 times! Each day the number of daily visits increases by 60%. Find the cumulative number of views over the first week.

(1) 102,061

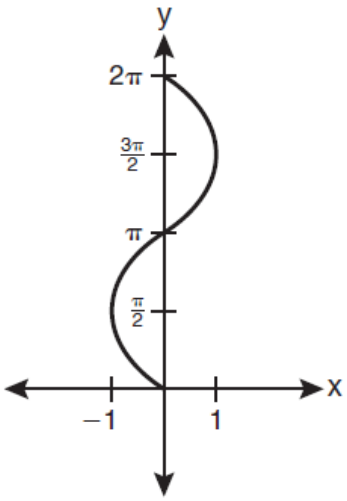
(2) 170,101

(3) 3,015,080

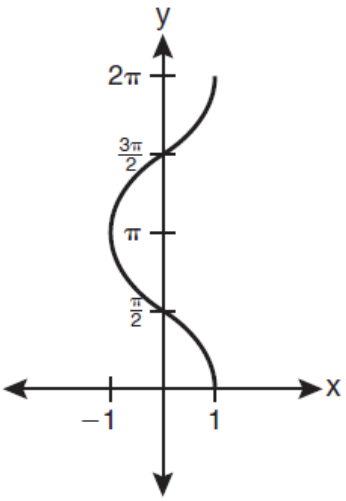
(4) 4,824,129

19. Which graph shows $y = \cos^{-1} x$?

Use this space for computations.



(1)



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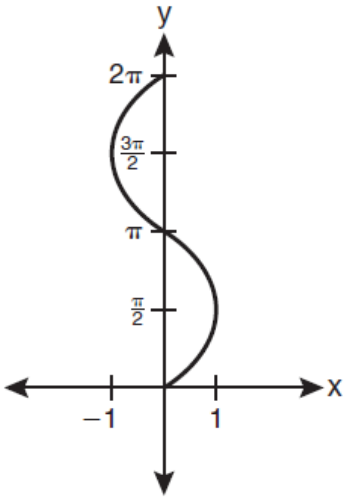

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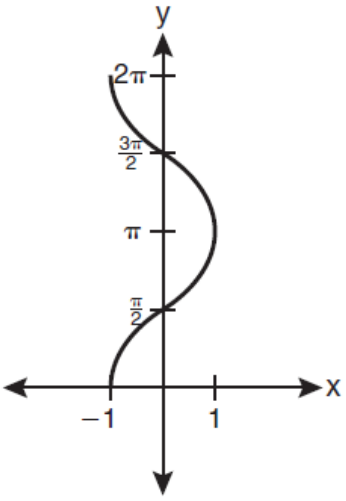


(4)

20. The solution set of $\sqrt{3x + 16} = x + 2$ is

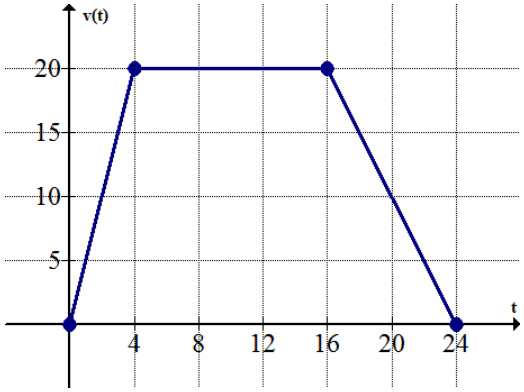
(1) $\{-3,4\}$

(2) $\{-4,3\}$

(3) $\{3\}$

(4) $\{-4\}$

21. A car is traveling on a straight road for $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise linear function defined by the graph below.



For $0 \le t \le 24$, which function represents the velocity of the car?

(1) $v(t) = \begin{cases} 5t, & 0 \le t \le 4 \\ -\frac{5}{2}t, & 16 \le t \le 24 \end{cases}$

(2) $v(t) = \begin{cases} 5t, & 0 \le t \le 4 \\ 20, & 4 < t < 16 \\ -\frac{5}{2}t + 60, & 16 \le t \le 24 \end{cases}$

(3) $v(t) = \begin{cases} 5t, 0 \le t \le 4 \\ 0t, & 4 < t < 16 \\ -\frac{5}{2}t + 60, & 16 \le t \le 24 \end{cases}$

(4) $v(t) = \begin{cases} 5t, 0 \le t \le 4 \\ 0t, & 4 < t < 16 \\ -\frac{5}{2}t, & 16 \le t \le 24 \end{cases}$

22. Tim correctly used a method of completing the square to solve the equation $x^2 + 7x - 11 = 0$. Tim's first step was to rewrite the equation as $x^2 + 7x = 11$. He then added a number to both sides of the equation. Which number did he add?

(1) $\frac{7}{2}$

(2) $\frac{49}{4}$

(3) $\frac{49}{2}$

(4) 49

23. The number of minutes students took to complete a quiz is summarized in the table below.

Minutes	14	15	16	17	18	19	20
Number of Students	5	3	x	5	2	10	1

If the mean number of minutes was 17, which equation could be used to calculate the value of x?

(1) $17 = \frac{119+x}{x}$

(2) $17 = \frac{119+16x}{x}$

(3) $17 = \frac{446+x}{26+x}$

(4) $17 = \frac{446+16x}{26+x}$

24. What is the radian measure of the smaller angle formed by the hands of a clock at 7 o'clock?

(1) $\frac{\pi}{2}$

(2) $\frac{2\pi}{3}$

(3) $\frac{5\pi}{6}$

(4) $\frac{7\pi}{6}$

$\frac{360}{12} = 30$

$5(30) = 150$

$150\left(\frac{\pi}{180}\right) = \frac{5\pi}{6}$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25. Express the product of $\left(\frac{1}{2}y^2 - \frac{1}{3}y\right)$ and $\left(12y + \frac{3}{5}\right)$ as a trinomial.

$$\left(\frac{1}{2}y^2 - \frac{1}{3}y\right)\left(12y + \frac{3}{5}\right)$$

$$6y^3 + \frac{3}{10}y^2 - 4y^2 - \frac{1}{5}y$$

$$6y^3 - \frac{37}{10}y^2 - \frac{1}{5}y$$

26. In a study of 82 video game players, the researchers found that the ages of these players were normally distributed, with a mean age of 17 years and a standard deviation of 3 years. Determine if there were 15 video game players in this study over the age of 20. Justify your answer.

$$P(\geq 20) = 0.158655259563$$

$$82(0.158655) = 13 \text{ video game players}$$

No, there are only 13 video game players over the age of 20.

27. Solve algebraically for all values of x:

$$\sqrt{x-5}+x=7$$

$$\sqrt{x-5}=7-x$$

$$(x-5)=(7-x)^2$$

$$x-5=49-14x+x^2$$

$$0=x^2-15x+54$$

$$0=(x-9)(x-6)$$

$x=9$, ✗
$x=6$✓

28. Evaluate $e^{x\ln y}$ when $x=3$ and $y=2$.

$$e^{3\ln 2}=8$$

29. If $f(x) = x^2 - 6$, find $f^{-1}(x)$.

$$x = y^2 - 6$$

$$x + 6 = y^2$$

$$\sqrt{x+6} = y$$

$$f^{-1}(x) = \sqrt{x+6}$$

30. Factor the expression $12t^8 - 75t^4$ completely.

$$3t^4(4t^4 - 25)$$

$$3t^4(2t^2 - 5)(2t^2 + 5)$$

31. Simplify the expression $\frac{3x^{-4}y^5}{(2x^3y^{-7})^{-2}}$ and write the answer using only positive exponents.

$$\frac{3x^{-4}y^5}{\frac{1}{4}x^{-6}y^{14}}$$

$$\frac{12x^2}{y^9}$$

32. Use an appropriate procedure to show that $x - 4$ is a factor of the function $f(x) = 2x^3 - 5x^2 - 11x - 4$. Explain your answer.

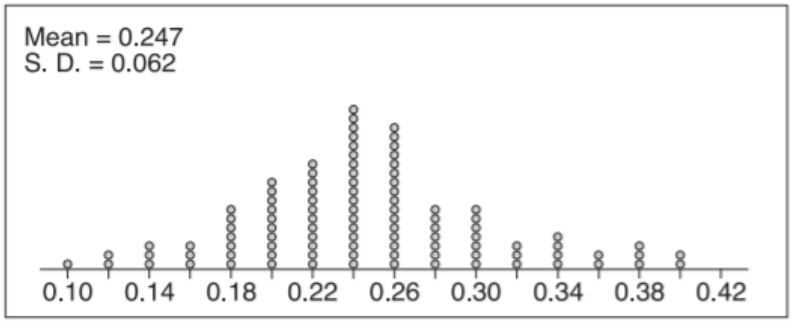
If $x - 4$ is the factor, then $x = 4$ is the root.
The reaminder theorem states that $x - 4$ is a factor, then $f(4) = 0$.

Since $f(4) = 0, x - 4$ is a factor of $f(x)$.

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33. Stephen's Beverage Company is considering whether to produce a new brand of cola. The company will launch the product if at least 25% of cola drinkers will buy the product. Fifty cola drinkers are randomly selected to take a blind taste-test of products A, B, and the new product. Nine out of fifty participants preferred Stephen's new cola to product A and B. The company then devised a simulation based on the requirement that 25% of cola drinkers will buy the product. Each dot in the graph shown below represents the proportion of people of preferred Stephen's new product, each sample size 50, simulated 100 times.



Proportion Preferring Stephen's Product

Assume the set of data is approximately normal and the company wants to be 95% confident of the results. Does the sample proportion obtained from the blind taste-test, nine out of fifty, fall within the margin of error developed from the simulation. Justify your answer.

Yes. The margin of error from this simulation indicates that 95% of the observations fall within ± 0.12 of the simulated proportion, 0.25. The margin of error can be estimated by multiplying the standard deviation, shown to be 0.062 in the dotplot. The interval 0.25 ± 0.124 includes plausible values for the true proportion of people who prefer Stephen's new product

The company decides to continue developing the product even though only nine out of fifty participants preferred its brand of cola in the taste-test. Describe how the simulation data could be used to support this decision.

The company has evidence that the population proportion could be at least 25%. The can expect to obtain a sample proportion of 0.18 or less several times despite the fact that the TRUE population proportion is 0.25. This, of course, is due to sampling variability. Given this information, the results of the survey do not provide enough evidence to suggest that the true proportion is not at least 0.25, so the development of the product should continue at this time.

34. Monthly mortgage payments can be found using the formula below:

$$M = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{n}}{\left(1+\frac{r}{12}\right)^{n}-1}$$

$M =$ monthly payment
$P =$ amount borrowed
$r =$ annual interest rate
$n =$ number of monthly payments

The Banks family would like to borrow \$120,000 to purchase a home. They qualified for an annual interest rate of 4.8%. Algebraically determine the *fewest* number of whole years the Banks family would need to include in the mortgage agreement in order to have a monthly payment of no more than \$720.

$$720 = \frac{120000\left(\frac{0.048}{12}\right)\left(1+\frac{0.048}{12}\right)^{n}}{\left(1+\frac{0.048}{12}\right)^{n}-1}$$

$$720 = \frac{480(1.004)^{n}}{(1.004)^{n}-1}$$

$$720(1.004)^{n} - 720 = 480(1.004)^{n}$$

$$-720 = -240(1.004)^{n}$$
$$3 = (1.004)^{n}$$

$$\log_{1.004}(3) = n$$

$$n = 275$$

$$275 \div 12 = 22.9 \text{ years}$$

It would take 23 whole years.

35. Solve the following system of equation algebraically for all values of $x, y,$ and z.

$$x + 3y + 5z = 45$$
$$6x - 3y + 2z = -10$$
$$-2x + 3y + 8z = 72$$

$x + 3y + 5z = 45$
$6x - 3y + 2z = -10$
$7x + 7z = 35$

$6x - 3y + 2z = -10$
$-2x + 3y + 8z = 72$
$4x + 10z = 62$

$-28x - 28z = -140$
$28x + 70z = 434$
$42z = 294$
$z = 7$

$4x + 10(7) = 62$
$x = -2$

$-2 + 3y + 5(7) = 45$
$y = 4$

$(-2,4,7)$

36. Write an explicit formula for a_n, the nth term of the recursively defined sequence below.

$$a_1 = x + 1$$

$$a_n = x(a_{n-1})$$

$a_2 = x(x + 1)$

$r = \frac{x(x+1)}{x+1} = x$

$a_n = (x + 1)(x)^{n-1}$

For what values of x would $a_n = 0$ when $n > 1$? $x = -1, 0$

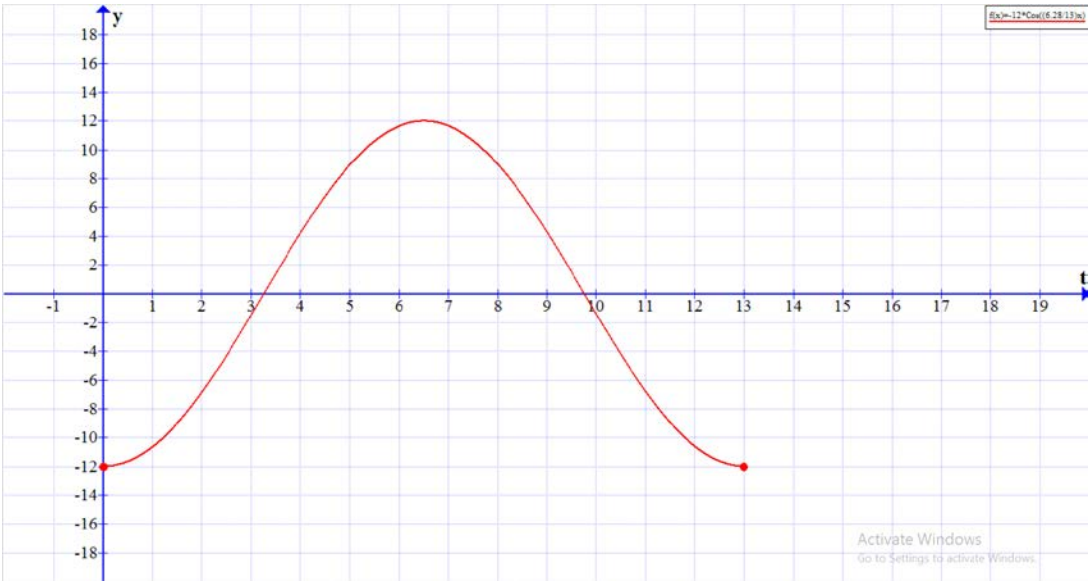
Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37. The ocean tides near Carter Beach follow a repeating pattern over time, with the amount of time between each low and high tide remaining relatively constant. On a certain day, low tide occurred at 8:30 a.m. and high tide occurred at 3:00 p.m. At high tide, the water level was 12 inches above local sea level; at low tide it was 12 inches below the average local sea level. Assume that high tide and low tide are the maximum and minimum water levels each day, respectively.

    Write a cosine function of the form $f(t) = A\cos(Bt)$, where A and B are real numbers, that models the water level, $f(t)$, in inches above or below the average Carter Beach sea level, as a function of the time measured in t hours since 8:30 a.m.

    On the grid below, graph one cycle of this function. $f(t) = -12\cos\left(\frac{-2\pi}{13}t\right)$



    People who fish in Carter Beach know that a certain species of fish is most plentiful when the water level is increasing. Explain whether you would recommend fishing for this species at 7:30 p.m. or 10:30 p.m. using evidence from the given context.

    According to our graph the tide is increasing from 8:30 am to 3:00pm. From 3:00pm to 9:30pm the tide is decreasing so I would not recommend fishing at 7:30pm. From 9:30pm to 4:00am the tide is increasing so I would recommend that they fish at 10:30pm.