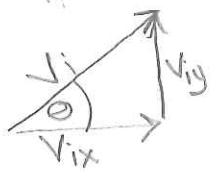
Topic 2C: Two Dimensional Projectiles

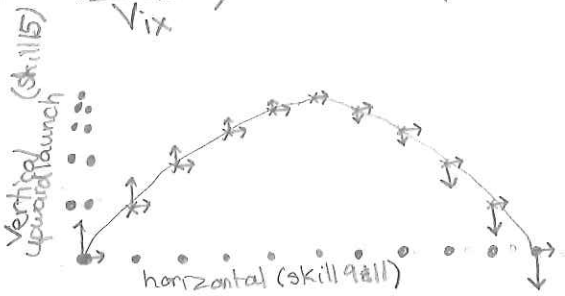
Skill 17: Projectiles launched at an angle

The initial velocity of an object launched at an angle must be broken down into horizontal velocity (v_{ix}) & vertical velocity (v_{iy})
(at equilibrium) (net force gravity)



$v_{ix} = v_i \cos\theta$
$v_{iy} = v_i \sin\theta$

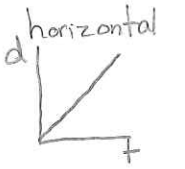
For angle less than 45° $v_{ix} > v_{iy}$
equal to 45° $v_{ix} = v_{iy}$
greater than 45° $v_{ix} < v_{iy}$

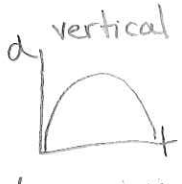


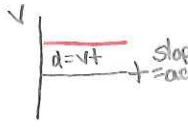
Vertical
$v_{iy} = v\sin\theta$
$a = -g = -9.81 m/s^2$
$v_{fy} = -v_{iy}$
total time $= \frac{-\Delta v}{-g} = \frac{-2v_{iy}}{-g}$
half time $= \frac{-v_{iy}}{-g}$ or $\frac{v_{fy}}{g}$
d_y for total time $= 0$
just path up / just fall $d_y = v_i t + \frac{1}{2}at^2$
$d_y = \frac{1}{2}at^2$

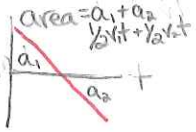
horizontal
$v_{ix} = v_i \cos\theta$
$d_x = v_x t$

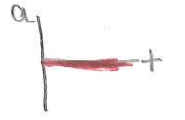
v_x	d_x	t
$v_i\cos\theta$		

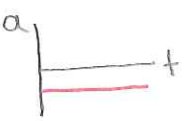












v_{iy}	Δv	d	a	t
$v_i\sin\theta$	$2v_{iy}$	0	$-9.81 m/s^2$	$\frac{2v_{iy}}{g}$

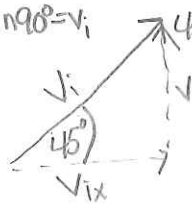
For total time of flight

For students who have completed Trig and are familiar with Trig IDs

$d_x = v_x t$ $v_x = v_i\cos\theta$
$t = \frac{2v_{iy}}{g} = \frac{2v_i\sin\theta}{g}$
so $d_x = (v_i\cos\theta)(\frac{2v_i\sin\theta}{g})$
$[2\sin\theta\cos\theta = \sin(2\theta)]$
so $d_x = \frac{v_i^2}{g}(\sin 2\theta)$

Skill 18: Launch angle and max d_y or d_x

90° $v_{iy} = v_i\sin\theta = v_i\sin 90° = v_i$
greatest v_{iy}, so greatest time of flight and max d_y



45° $v_{ix} = v_{iy}$
v_{iy} greatest horizontal range d_x

- Angle of 45° or closest to 45° with have greatest range
- angles equal distance from 45° will land in same place ie 40°&50°, 30°&60°, 10°&80°

0°
$v_{iy} = v_i\sin 0°$
$v_{iy} = 0$
horizontal projectile

trig fact to remember

$\cos 0° = 1$	$\sin 90° = 1$
$\cos 60° = .5$	$\sin 30° = .5$
$\cos 30° = \sin 60° =$	
$\cos 45° = \sin 45° =$	
$\tan 45° = 1$	

$\frac{\sin}{\cos} = \frac{\frac{o}{h}}{\frac{a}{h}} = \frac{o}{a} = \frac{A_y}{A_x} = \tan\theta$

- as angle increases from 0 to 90°
  - $\cos\theta$ decreases, so does v_x
  - $\sin\theta$ increases, so does v_{iy}
    - time of flight
    - max height
  - d_x is a result of v_x & time which depends on v_y

So d_x is max when v_x & v_y are equal