UNIT ONE

Topic 1A: Mathematical Reasoning

Skill 1: Translating between scientific, metric and standard notation

Lots of options exist for expressing a measurement or numerical answer. In order to recognize the many forms of an answer you need to understand a little background on each format.

- **Standard notation** – Long form of a number with base units. Ex – 23,400,000 meters or 0.000045m (ie write out zeros as place holders)
- **Scientific notation** – Only significant figures* are stated in the coefficient and power of 10 is replaced by an exponent (10^n). A positive exponent means multiply by 10 "n" times. A negative exponent means divide by 10 "n" times.
  - Example $4.5 \times 10^6 = 4.5 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 4{,}500{,}000$
    $4.5 \times 10^{-6} = 4.5/10/10/10/10/10/10 = 0.0000045$
- **Metric notation** – A metric prefix replaces 10^n. Prefixes each have symbols as shown in the chart. The prefixes are added to base units for
  - Length (meters)
  - Mass (grams)
  - Time (seconds)
  - Charge (Coulombs)

Metric prefixes can also be added to derived units such as the Newton $\frac{(kilograms)(meters)}{seconds^2}$; Joules $(kg\frac{m^2}{s^2})$ etc.

*The terms **significant figures** (SigFigs) refers to level of accuracy in measurement. For our purposes it is similar to degree of rounding

Exponent	Number of time to multiply or divide	Multiplier	Metric prefix	Metric symbol
10^9	1 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x10	1000000000	Giga	G
10^6	1 x 10 x 10 x 10 x 10 x 10 x 10	1000000	Mega	M
10^5	1 x 10 x 10 x 10 x 10 x 10	100000		
10^4	1 x 10 x 10 x 10 x 10	10000		
10^3	1 x 10 x 10 x 10	1000	kilo	k
10^2	1 x 10 x 10	100		
10^1	1 x 10	10		
10^0	1	1		
10^{-1}	1 /10	0.1	deci	d
10^{-2}	1 /10 /10	0.01	centi	c
10^{-3}	1 /10 / 10 /10	0.001	milli	m
10^{-4}	1 / 10 /10 /10 /10	0.0001		
10^{-5}	1 /10 / 10 / 10 / 10 / 10	0.00001		
10^{-6}	1 /10 / 10 / 10 / 10 / 10/ 10	0.000001	micro	μ
10^{-9}	1 /10 / 10 / 10 / 10 / 10/ 10/ 10/ 10/ 10	0.000000001	nano	n

In order to recognize terms expressed in different forms or to combine values given in various forms you must be able to efficiently move between systems.

General rule: Any change in the coefficient is balanced by an opposite change in exponent or unit (and vice versa)

Working with Scientific notation

2.03×10^{3} ← Exponent

Coefficient ↑ Base ↑

Proper scientific notation requires that Coefficients must be greater than or equal to 1 but less than 10

$1 \leq coefficient < 10$

Operations in Scientific Notation (aka SN)

Multiplying scientific notation

Multiply coefficients and add exponents (combine units algebraically, ie simplify.)

$(2 \times 10^{4}\ s)\ (3 \times 10^{8}\ m/s) = (2 \times 3)\ (10^{4+8})\ (s)(m/s) = 6 \times 10^{12}\ m$

Dividing in scientific notation

Divide coefficients and subtract exponents (combine unit algebraically)

$(7 \times 10^{8}\ m) / (2 \times 10^{3}\ s) = (7/2) \times (10^{8-3})\ (m/s) = 3.5 \times 10^{5}\ m/s$

Converting between powers

When translating between forms you may need to express values without proper scientific notation in order to combine terms or to express with metric notation.

Any change in the exponent must be balanced by a change in the decimal placement of the coefficient and vice versa.

2.03×10^{3} is the same as

- 20.3×10^{2} think of it as $(2.03 \times 10^{1} \times 10^{2})$
- 0.203×10^{4} think of it as $(2.03 \times 10^{-1} \times 10^{4})$

- If the coefficient gets larger (decimal to the right) the exponent gets smaller by the same number of places.
- If the coefficient gets smaller (decimal to the left) the exponent gets larger by the same number of places.

Working with metric prefixes

Metric prefixes replace exponents:

- For powers that match a prefix simply replace "$x10^{n}$" with the appropriate prefix combine with base unit: 7×10^{3} m = 7 km

- For powers that do not match a common prefix, convert to nearest prefixes and put the remaining value in standard or scientific notation
  [10^{6} (Mega), 10^{3} (kilo), 10^{-2} (centi), 10^{-3} (milli) 10^{-6} (micro), 10^{-9} (nano)] see PRT for the rest

$0.0005m = 5 \times 10^{-4}\ m = 0.5 \times 10^{-3} m = 0.5mm$ or $5 \times 10^{-1} mm$

Or $= 500 \times 10^{-6} m = 500\mu m$ or $5 \times 10^{2}\ \mu m$

Notice that multiplying exponents matches the starting value