

Skill 17 – Projectiles launched at an angle

A projectile launched at an angle experiences a momentary force from a launching device and then experiences only the force of the gravitational field from launch to impact.

3 skills are combined to solve projectile at angle problems		
Determine horizontal and vertical components of initial velocity (Unit 1 – Skill 10)	Analyze VERTICAL motion for upward launched projectile (Unit 2- Skill 4 and 5)	Analyze HORIZONTAL motion at equilibrium (Unit 1- Skill 9)
EQUATIONS $v_{ix} = v_i \cos \Theta$ $v_{iy} = v_i \sin \Theta$ $\Theta = \tan^{-1}\left(\frac{v_{iy}}{v_{ix}}\right)$ $v_i^2 = v_{ix}^2 + v_{iy}^2$	EQUATIONS $d = v_i t + \frac{1}{2} a t^2$ $v_f^2 = v_i^2 + 2ad$ $v_f = v_i + at$	EQUATIONS $d = \bar{v} t$
The initial velocity (hypotenuse) is not an "x" or "y" vector and must be broken down into components because the gravitational field only acts on the vertical axis. The horizontal velocity does not experience a force and therefore remains unchanged.	The initial vertical velocity is positive and the acceleration is opposite (negative). Remember that the high point aligns with $\frac{1}{2}$ time, velocity equals zero.	The horizontal velocity does not experience a force.

Example:

What is the horizontal range and maximum vertical displacement of a projectile launched with a velocity of 20 m/s at an angle of 30 degrees?

	Horizontal	Vertical
Step 1: Determine the horizontal and vertical components of the initial velocity	$v_{ix} = v_i \cos \Theta$ $v_{ix} = 20 \text{ m/s} (\cos 30^\circ)$ $v_{ix} = 20 \text{ m/s} (\cos 30^\circ)$ $v_{ix} = 17.3 \text{ m/s}$	$v_{iy} = v_i \sin \Theta$ $v_{iy} = 20 \text{ m/s} (\sin 30^\circ)$ $v_{iy} = 10 \text{ m/s}$
Step 2: Use kinematics equations to solve for unknowns using rules for each axis for total time of flight	$a = 0$ $d = \bar{v}_x t$ $d = (17.3 \text{ m/s})(2.04 \text{ s}) = 35.3 \text{ m}$	$a = g = -9.81 \text{ m/s}^2$ $v_f = v_i + at$ $-10 \text{ m/s} = 10 \text{ m/s} + (-9.81 \text{ m/s}^2)t$ $t = \frac{-20 \text{ m/s}}{-9.81 \text{ m/s}^2} = 2.04 \text{ s}$
Step 3: Use kinematics equations to solve for height at mid-way point		$t = 1.02 \text{ s}$ Height up equals height down. Solve for falling half to keep problem simple $v_{iy} = 0$ use shortcuts $d = \frac{1}{2} a t^2 = \frac{1}{2} (9.81 \text{ m/s}^2) (1.02 \text{ s})^2$ $d = 5.1 \text{ m}$