

SKILL 21 – WORKING WITH FORCE VECTORS

Force is a quantity in which both the magnitude (size) and direction are important. In other words, **FORCE IS A VECTOR. ALL FORCES ARE MEASURED IN NEWTONS** which can also be expressed as $(\text{kg}\frac{\text{m}}{\text{s}^2})$

In order to find the NET FORCE (F_{net} or ΣF) acting on an object you must be able to identify all the forces acting on an object and add them up according to the rules we've learned for vectors. The unit of force is the NEWTON (N) which is the same thing as a $\text{kg}\frac{\text{m}}{\text{s}^2}$ this comes from the equation $\Sigma F = ma$ (replace the variables with the appropriate units).

Review Skill - Combining Vectors:

THE SUM OF TWO VECTORS IS CALLED THE RESULTANT. IN ORDER TO DETERMINE THE RESULTANT OF TWO VECTORS FIND THE DIFFERENCE IN DIRECTION (assigned angle) BETWEEN VECTORS.

THE SAME DIRECTION MEANS (0 degrees difference). FORMS THE MAXIMUM RESULTANT.

OPPOSITE DIRECTION MEANS (180 degrees difference). FORMS THE MINIMUM RESULTANT.

THE POSSIBLE RANGE OF RESULTANTS DEPENDS ON THE ANGLE BETWEEN THE VECTORS.

Remember, in order to add vectors you must pay attention to the axis on which they act.

- A. **Vectors on the same axis:** If the FORCE vectors fall on the same axis they can be added with directions assigned. (Write the equation as addition and substitute in negative values).

Ex: 15N right, 20N left and 15N right = $15\text{N} + (-20\text{N}) + 15\text{N} = 10\text{N}$ or 10N right

10 N North, 15N South and 5 N North = $10\text{N} + (-15\text{N}) + 5\text{N} = 0\text{N}$

- B. **Vectors on perpendicular axes:** Once you determine the FORCE ON EACH AXIS ($F_{x\text{net}}$, ΣF_x or $F_{y\text{net}}$, ΣF_y) you then must apply the general equations $A_x = A\cos\theta$, $A_y = A\sin\theta$, $\theta = \tan^{-1}(A_y/A_x)$ and $A^2 = A_x^2 + A_y^2$ which can now be written as

$$F_x = \Sigma F\cos\theta, \quad F_y = \Sigma F\sin\theta, \quad \theta = \tan^{-1}(\Sigma F_y / \Sigma F_x) \quad (\Sigma F)^2 = \Sigma F_x^2 + \Sigma F_y^2$$

Ex: A force of 15N east and a force of 30N north act concurrently on an object. What is the resultant force acting on the object?

$$(\Sigma F)^2 = \Sigma F_x^2 + \Sigma F_y^2 \quad \theta = \tan^{-1}(\Sigma F_y / \Sigma F_x)$$

$$F = \sqrt{F_x^2 + F_y^2} \quad \theta = \tan^{-1}(30\text{N}/15\text{N})$$

$$F = \sqrt{15\text{N}^2 + 30\text{N}^2}$$

- C. **Vectors that don't fall on either the vertical or horizontal axis** must be broken down or "resolved" using the equations above before being considered in the equation.

Ex: Force A is 10N @ 45 degrees and Force B is 20 N at 30 degrees. What is the resultant of Force A and Force B.

$$\begin{aligned} \Sigma F_x &= F_{Ax} + F_{Bx} \\ F_{Ax} &= F_A \cos\theta \text{ and } F_{Bx} = F_B \cos\theta \\ \Sigma F_x &= F_A \cos\theta + F_B \cos\theta \\ \Sigma F_x &= 10\text{N}\cos 45^\circ + 20\text{N}\cos 30^\circ \\ \Sigma F_x &= 10\text{N}\cos 45^\circ + 20\text{N}\cos 30^\circ \end{aligned}$$

$$\begin{aligned} \Sigma F_y &= F_{Ay} + F_{By} \\ F_{Ay} &= F_A \sin\theta \text{ and } F_{By} = F_B \sin\theta \\ \Sigma F_y &= F_A \sin\theta + F_B \sin\theta \\ \Sigma F_y &= 10\text{N}\sin 45^\circ + 20\text{N}\sin 30^\circ \\ \Sigma F_y &= 10\text{N}\sin 45^\circ + 20\text{N}\sin 30^\circ \end{aligned}$$

$$\begin{aligned} (\Sigma F)^2 &= \Sigma F_x^2 + \Sigma F_y^2 \\ \theta &= \tan^{-1}(\Sigma F_y / \Sigma F_x) \end{aligned}$$

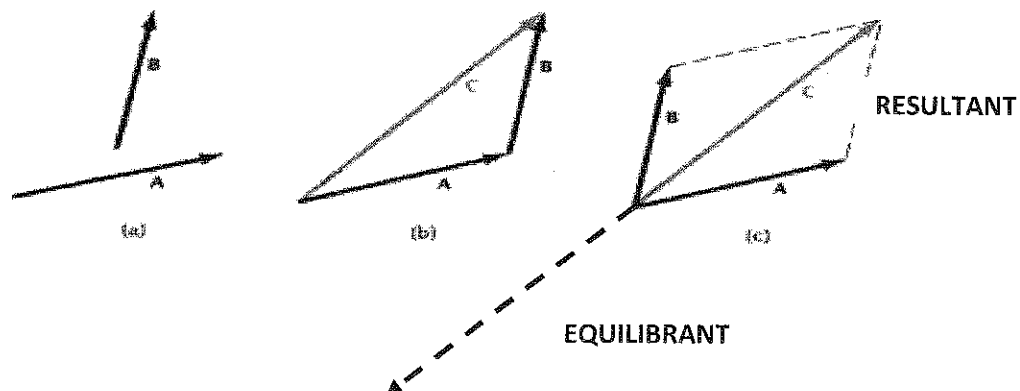
D. Adding Scaled Force Vectors using Graphical Methods ("Head to Tail" or "Tail to Tail")

We have 2 methods of adding vectors

Head to Tail Method

"Tail to Tail" Method

Parallelogram Method



The force required to balance a resultant of two or more vectors is called the **EQUILIBRANT**. The **EQUILIBRANT** is equal in magnitude and opposite in direction (180 degrees away) from the **RESULTANT**. Equilibrium means acceleration is zero since $F_{\text{net}}=0$.