Skill 7: Graphical (scaled) addition of two dimensional vectors

This skill can be applied to vectors on any axis but is most useful for vectors that are not on the same axis. Remember that all of these methods are tools for combining motion two or more of the same type of measurement in which direction must be included in the value.

The sum of two or more vectors is called the RESULTANT

Head to Tail addition (adding consecutively)

If given a list of vectors:

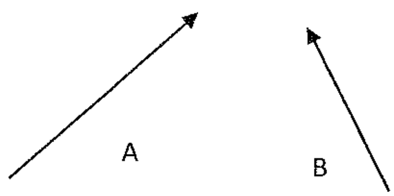
- Choose a starting vector and draw to scale with angle relative to zero degrees (east). Include arrow at end.
- Place the tail of the next vector at the end of the first and draw to scale with angle relative to east (not the prior vector).
- Repeat with all subsequent vectors

Tail to tail addition (adding concurrently)

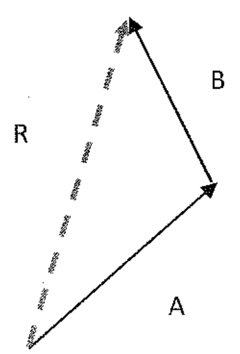
Used for combining two vectors at a time.

- Draw both vectors to scale with both magnitude and direction from a common point. $(\vec{A} + \vec{B})$
- Create a parallelogram by replicating each vector to scale at the head of the other vector. (ie $(\vec{A} + \overrightarrow{B'})$ and $(\vec{B} + \overrightarrow{A'})$

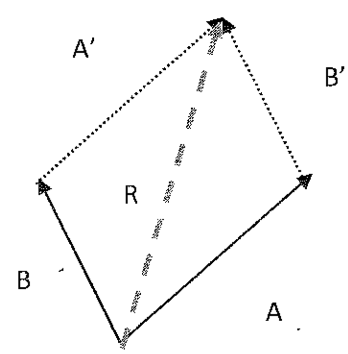
Examples:



Head to Tail



Tail to Tail (Parallelogram)



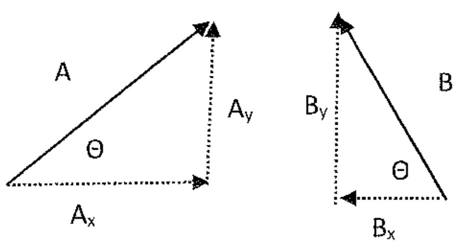
REMEMBER THE RESULTANT MAGNTITUDE AND DIRECTION SHOULD MAKE SENSE FOR THE COMBINED MOVEMENT:

If Vector A is described as UP and to the RIGHT and by comparison Vector B is equally UP and a little to the left, the combined resulting vector would be roughly twice the height and not as far right as vector A.

Skill 8: Vector equations for perpendicular vectors
The following equations can be used to find the horizontal and vertical components of a resultant vector or vice versa to combine two perpendicular components into a single resultant.

In the prior skill we described two vectors by the component horizontal and vertical motion.
A is a vector that is to the right (Ax) and up (Ay); B is a vector that is a little to the left (Bx) and up (By).
This is done for 2 reasons.

- Right triangles are easy to work with using Pythagorean Theorem and SOH CAH TOA
  - Hypotenuse (H) = Resultant A, B
  - Adjacent (A) = horizontal component Ax, Bx
  - Opposite (O) = vertical component Ay, By
  - Angle (Θ) "theta" = Angle between horizontal component and hypotenuse

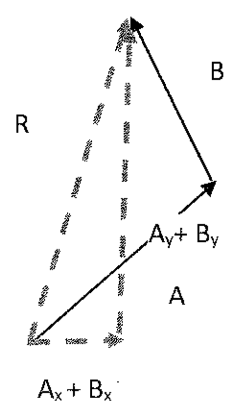


In order to solve for other sides for any vector triangle you must know 2 of 4 possible values: A, Ax, Ay or Θ and choose an equation from the following list to solve:

Ax = A cos Θ   Ay=AsinΘ $\Theta = \tan^{-1}(\frac{A_y}{A_x})$

$A^2 = A_x^2 + A_y^2$ or $A = \sqrt{A_x^2 + A_y^2}$

- To find the sum of the resultants A and B you can add components Ax + Bx to get total horizontal (Rx) and Ay + By to get total vertical (Ry)



Example:
If a dog walks 30 m east and 50m north the resultant displacement of the dog would be found by:

Ax=dx= 30m   Ay=dy= 50m   A=d=?   direction (angle) =Θ=unknown

So $A = \sqrt{A_x^2 + A_y^2}$ $A = \sqrt{(30m)^2 + (50m)^2}$ = 58.3m

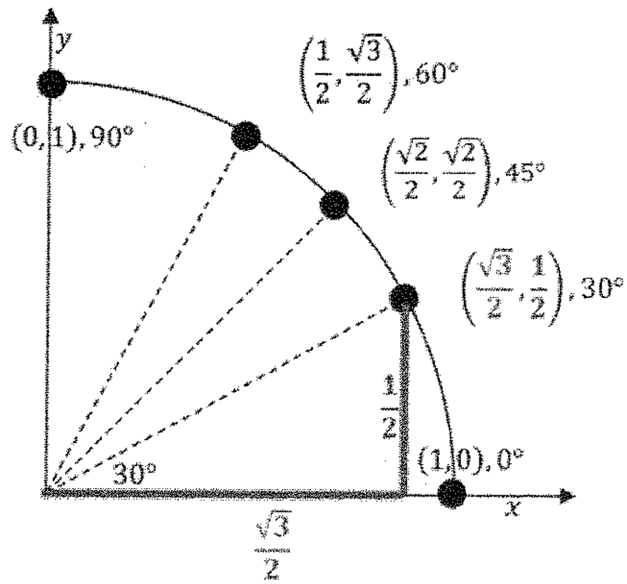
and $\Theta = \tan^{-1}(\frac{A_y}{A_x})$ = $\Theta = \tan^{-1}(\frac{50m}{30m}) = 59\ degrees$

Vectors have both a magnitude and direction so the resultant is

58.3m @59^0

TRIG REFERENCE FOR 0 to 90 degrees

The diagram below shows the ratio of the horizontal and vertical components for a constant radius of "1" at various angles. While the numbers given here are reported in radical form, answers for the Physics Regents exam are written as decimals or in scientific notation. (See chart below)



In order to use this chart to estimate resultants based on known components or vice versa, remember to follow the format of "x" and then "y" where the angle Θ is measured between the horizontal component and the resultant.

	Sin	Cos	Tan	x vs y
Equation	$\sin\Theta = \frac{opp}{hyp}$	$\cos\Theta = \frac{adj}{hyp}$	$\tan\Theta = \frac{opp}{adj}$	
0^0, East	0	1	0	All x
30^0	$\frac{1}{2}$; 0.5	$\frac{\sqrt{3}}{2}$; 0.866	$\frac{\sqrt{3}}{3}$; 0.577	x>y
45^0, Northeast	$\frac{\sqrt{2}}{2}$; 0.707	$\frac{\sqrt{2}}{2}$; 0.707	1	x=y
60^0	$\frac{\sqrt{3}}{2}$; 0.866	$\frac{1}{2}$; 0.5	$\sqrt{3}$; 1.732	x<y
90^0, North	1	0	Undefined	All y

The above can be modified to any quadrant as long as angles are measured relative to the positive or negative x axis (ie East or West)

Quadrant One (+x, +y) 0 to 90 degrees
Quadrant Two (-x, +y)
Quadrant Three (-x, -y)
Quadrant Four (+x, -y)